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Model of identification of optimal debt structure in corporations

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Concentration — Master in Corporate Finance

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ЗАЯВЛЕНИЕ О САМОСТОЯТЕЛЬНОМ ХАРАКТЕРЕ ВЫПОЛНЕНИЯ

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Описание цели, задач и основных результатов	Вопрос структуры капитала корпораций является широко обсуждаемым в академической и бизнес-среде, так же как и идентификация оптимальной структуры капитала. В данном случае целью исследования было построение модели идентификации оптимальной структуры капитала. Разработка модели производилась на базе модели с подходом к собственному капиталу компании как к реальному опциону. Результатом диссертации стало построение модели идентификации оптимальной структуры капитала и разработка алгоритма применения этой модели к реальным компаниям. Разработанная модель была применена для реальных компаний, находящихся в разных экономических условиях. Диссертация имеет теоретическую и практическую значимость, поскольку с одной стороны модель была теоретически обоснована и были даны предпосылки к дальнейшему направлению исследований в данной области, с другой стороны, предложенная методика применения данной модели была протестирована на реальных компаниях и были даны практические рекомендации.
Ключевые слова:	Структура капитала, оптимальная структура капитала, оптимальная структура долга, валютный риск долга, риск дефолта, оптимальный леверидж

ABSTRACT

Master Student's Name	Anton G. Solovev
Master Thesis Title	Model of identification of optimal debt structure in corporations
Faculty	Graduate School of Management (SPbSU)
Main field of study	Corporate Finance
Year	2016
Academic Advisor's Name	Alexander V. Bukhvalov
Description of the goal, tasks and main results	The problem of corporations' capital structure is widely discussed in the academic and business environment, as well as the problem of identification of the optimal capital structure. In this case, the purpose of the study was to construct a model of identifying the optimal capital structure. Development of the model was made on the basis of a model with an approach to the equity of the company as a real option. The result of the thesis were the built model identifying the optimal capital structure and the algorithm used to apply this model to real companies. The developed model was applied to real companies in different economic conditions. The thesis has theoretical and practical significance, since, on the one hand, the model was theoretically substantiated and were given the prerequisites for the further direction of research in this area, on the other hand, the proposed method of application of this model has been tested on real companies and practical recommendations were made
Keywords	Capital structure, optimal capital structure, optimal debt structure, currency risk of debt, default risk, optimal leverage

TABLE OF CONTENTS

INTRODUCTION

Background and actuality of the study. Optimal capital structure – is the capital structure that is chosen in a way, in which the company maximizes its overall value, since the maximizing value for the stakeholders can be called the main goal for every company. But in the same time optimal capital structure also must minimize the overall cost of capital for the company. There are a huge variety of approaches to identification the optimal capital structure. However, most of them have certain limitations and there is no common agreement in the scientific and business society, what model should be generally used. This explains the actuality of the research, since there are plenty of works published every year concerned this topic, which indicates the relevance and importance of the topic for the business and scientific societies, but there is still no commonly accepted model. Further literature review gives an evidence of the primarily theoretical character of the current general models and approaches of optimal debt structure identification. In spite of plurality and variety of this type of models, most of them have a lot of assumptions that are hardly suitable for use in the cases of real companies. Alternatively, plenty of practically oriented works in the volume of interest are devoted to examination of specific situations or markets that limits their potential for more common use.

The models of identifying the optimal capital structure can be divided to the two big groups. The first one, are the models based on the Modigliani-Miller's theory, and they can be called classical models. However, despite the fact that classical optimal structure models are widely used and discussed in academic papers, they often require unrealistic assumptions that make them not practically valuable. Moreover, the models based on Modigliani-Miller's theory do not fully and accurately describe the relationship between required return on equity capital and risk of default, because in the situation of company's default even if shareholders require incredibly high return, they will not get it eventually.

However, this thesis is aimed at building an applicable model that may be analyzed and leads to the certain practical implications. That is why this paper is focused on the other type of optimal capital structure models defining equity as an option. Those types of models predicting the best capital structure using Black–Scholes–Merton differential equation. This type of models seems to have more realistic approach. Nevertheless, these models are really complicate and hard to modify, because mostly pure mathematics approach is used in them. So, there is not really wide range of the works, that implement this models on practice and analyze the gained results. In the following work, one of the models with option-like approach was chosen as a basis model for the further modification and implementation. This also explains actuality and relevance of the study, due to the fact that the application of the option-like models to the real cases is not so well

learned as classical models. As the base model was chosen the model presented by Leland in 1994, due to the interesting approach to the debt values, and the fact that from all option-like models it has the best balance of possibility to be modified, and realistic approach.

Managerial implications of the model seem to be very broad. Interest of the managers of the companies to the research is often due to possibility of optimal debt structure determination for their companies. Moreover, this work should be relevant for different kinds of investors by virtue of opportunity to predict the kind of debt of the companies they wish to invest, understand the amount of risk that company holds and it's perspectives to grow in terms of efficient using of financial leverage. As supplementary to the aforementioned applications, kind of model developed may be a useful tool for banks and other similar organizations in the perspective of either clients' risk identification or targeting their services by offering special kinds of debts to companies.

The objectives and research methodology. The research goal can be stated as developing the optimal debt structure identification model. Practical focus of the research goal provides an advantage to this paper as against existing studies in examined field of expertise. Thereby, the research goal of this paper can be attributed to the aspiration of developing the universally applicable model with the capability of implementation in the real cases, which might be suitable for the wide range of real companies.

The subject of the research is the company's capital structure, which shows the balance between only long-term sources of funding, i.e. balance between debt and equity. Whereas the object of the research is the "optimal" capital structure, which can be described as chosen capital structure from all possible ones, which creates the biggest value for the company's stakeholders.

The objectives of the paper represent the sequence of actions in place to reach the research goal, specified above:

- to analyze the existing models of optimal debt structure identification;
- to identify factors that can determine capital structure;
- to examine risks that can be mitigated by capital structure;
- develop debt structure management model for optimization of capital structure
- to establish a standard optimal capital structure calculation procedure;
- apply developed model to real firm and analyze the results

Outline of the study. The thesis is divided on the tree chapters. In the first chapter, firstly the concept of the capital structure will be introduced. Then, the existing models of capital structure optimization will be analyzed in order to achieve the first objective of the thesis, and partly achieved second and third objectives.

In the second chapter firstly the base theoretical model will be described, thus the second and the third objectives will be fully achieved. Then the process of the model development will be described and the modifications that were made to the base model. Then, the standard optimal capital structure calculation procedure will be established, so the fourth thesis objectives will also be achieved.

In the third chapter, the real companies, on which the model will be tested will be described, with the analysis of the market conditions, on which these companies operates. So the last objective will be achieved in the third chapter, as well as the goal of the research. Also, main managerial and theoretical contributions of the current research will be presented.

1. CONCEPT OF THE CAPITAL STRUCTURE AND EXISTING MODELS OF CAPITAL STRUCTURE OPTIMIZATION

1.1. Capital structure

This paragraph describes advantages and inconveniences of each source of capital; furthermore, conceptions of capital structure and structure of sources of financing are explained; relationship between increase in debt and equity risk and return (financial leverage concept) is analyzed; financial, commercial risks and taxes are described as main factors reflecting the choice of capital structure; moreover, the key capital structure concepts that seek for the optimal structure of capital (maximizing its value) are explained and their pros and cons are given; finally, the key concepts of optimal capital structure are introduced and explained.

Financial management decisions may be divided into two main directions: investment decisions that describe funds invested in real or financial assets, and financial decisions that consist in a choice of sources of current and fixed financial assets. The second direction reflects the conception of capital structure decisions, dedicated to finding a balance between equity and debt. Suggesting the only firm activity is a continuous process of investments projects implementation, it may be stated that financial decisions are all about funding investment projects and choosing the sources of financing. Here occurs the dilemma – a need to choose a proper balance between different sources of financing that distinct in their costs because of tradeoff between risk and return Moreover, finding a proper balance between sources may increase the value of the capital itself.

Financial decisions are divided into short-term and long-term, defining the goal of funding – current or longer-term needs in assets. The key point of long-term financial decisions is the choice of the combination of borrowed capital and equity that would maximize the total value of capital $V = E + D$, where E – is a market value of equity and D – is a market value of debt. The problem of choosing a proper (or optimal as stated in different sources) combination of debt and equity is widely disputed and requires a closer look that will be described later.

The structure of financing sources is the balance between different short-term and long-term sources. The capital structure shows the balance between only long-term sources of funding, i.e. balance between debt and equity. The main difference between debt and equity is about the risk level and consequent required return of the owner of capital.

Cost of capital

The key condition that influences the choice of using a particular type of resources is its price or costs. This condition is also related to the financial resources. Different components of financial resources have different costs as obtained from different marketplaces: money market,

stock market or commodity market. The costs of financial resources are also explained by maintenance price. The relative costs on maintaining different elements of capital is *the cost of capital*. The cost of capital of a certain resource may be found as shown in the formula1.1.

$$Cost\ of\ capital = ¿\ the\ capital\ owner\ \frac{Payments\ made\ ¿}{Capital\ attracted}, (1.1)$$

The cost of capital is strongly related to the return required by the owner of capital. Generally, owners of capital require bigger returns on riskier assets (in terms of the companies' activities) and, therefore, the cost of capital is also larger than in the case of less riskier assets. In the conditions of competitive market, owners of capital may choose and relate the risk and return of different investment assets. The cost of capital is defined as current risk-free return on investments, estimated pace of inflation and risk premium. The owner of capital observes different options of investments on different markets and chooses a certain return on investments in accordance with the risk implied. Making financial decisions, financial manager evaluates the cost of each element of the capital and the total cost of capital as combination of these elements. The total cost of capital of the corporation is often used as a discount rate for the present value of future cash flows. To understand the total cost of capital of a company financial manager use Weighted Average Cost of capital (WACC) showed in the formula 1.2.

$$WACC = \frac{r_{debt} * (1 - r_{tax}) * Debt}{Debt + Equity} + r_{equity} \frac{Equity}{Debt + Equity}, (1.2),$$

where r_{debt} – is the debt interest rate; r_{equity} – is sharehoders' required rate on return; and r_{tax} – is the tax rate on companies' profit.

Evaluation and comparison of different elements of the capital allow to choose the most cheap way of long-term financing, in other words to choose the target structure of capital. The cost of capital differs not only between different elements, but also into different conditions implied on each of the elements. For example, required return and, correspondingly, risk of each element of the capital change over time, that leads to the changes in cost of capital.

Borrowed capital

The main advantage of the borrowed capital is the low costs associated with this source, in comparison with equity. This may be explained by different factors:

- Maintenance costs of the debt are loer than for equity as it does not require registrator services or underwriting services;

- Before tax interest rate on debt is lower that required rate of return for equity as the risk is lower (because in the case of the company's default requirememts of the debt holders will be satisfied first);
- Debt payments make the tax base lower, this effect os called *tax shield*;
- Debt-holders do not have rights to run the company and it does not imply risk of firing for managers;

Relatively to the equity, borrowed capital has wide range of opportuninties to attract the capital. Moreover, the debt may provide a good potential to financial growth, as it enables growth of profitability and return on equity.

On the other hand, borrowed capital generates risk of decrease in financial stability and sovency. That is why the main disadvantage of this type of capital is exessive risk for the equity-holders, because in this case shareholders need to suffer additional risks besides commercial – financial risk. As the result, shareholders increase required rate of return as the additional risk compensation. In this way, borrowed capital may be attractive for its cheapness, but it is also associated with additional shareholders' risks.

Another disadvantage of borrowed capital is related to the need of money concentration by the time loans have to be paid off. As the loan repayment term is accurately defined, management of a company has to work out special repayment schemes. Moreover, companies may cover their debts with issuing new capital.

Despite the fact that creditors do not have rights to run a company, in a situation when the share of debt is significant, debt-holders may control some of the company's decisions. In some cases in bank loan agreement may be marked a mandatory share of net income retention that may contradict with the managers' interests. Furthermore, guarantee of pledge are often required. If company's shares serve as collateral, in a case when company is unable to serve the debt, creditors may take control over the company. This situation took place during the crisis of 1998 in Russia. Companies may occur in a more difficult situation if they provide theur currency export earnings as the collateral, because in this case the need of repaying the debt at the same time being unable to issue the new one.

Equity

Generally, equity may be presented as difference between total assets and liabilities, as an accounting measure. However, under equity the market price of issued stock is usually understood.

The main advantage of this source of capital is the level of financial responsibility towards shareholders. Companies are not obliged to make regular interest payments and may

redirect their cash flows on the business growth; furthermore, if business is not successful, shareholders are those who take the hit, because creditors are the first who receive contribution.

However, because of higher risk shareholders take they may take control over the managers' decisions. Furthermore, the share issue is costly and requires a lot of time; that is why it is not rational to finance separate projects with new share issues, because equity is a longer-term source of capital that debt.

1.2. Theoretical review of existing models of capital structure optimization

Optimal capital structure

Optimal capital structure – is the capital structure that is chosen in a way, in which the company maximizes its overall value, since the maximizing value for the stakeholders can be called the main goal for every company. But in the same time optimal capital structure also must minimize the overall cost of capital for the company. The problem here, is that when company is taking more debt, thus increase debt to equity ratio, its overall value is increasing, due to benefits of the debt, most important of which is tax shield. However, when the company is increasing its debt levels, the risk of the company is also getting higher, since it have to repay the debts, and at one point it the repayments of debt might become so high, that the company will have to call themselves a bankrupt. Due to this fact, the riskiness of the firm is rising with rising debt to equity ratio, which is also measured by increased overall cost of capital. So, the point here is to find a balance between these two trends is the main question of finding the optimal capital structure.

1.2.1. Classical theories of the optimal capital structure

Modigliani-Miller model

By the end of 1950s the theory of the capital structure had not existed itself. In the year 1958 F. Modigliani and M. Miller published the article "The Cost of Capital, Corporation Finance and the Theory of Investment" [Modigliani, Miller, 1958], where the basics of capital theory structure were laid. The main point in their theory is that the firm value does not depend on the capital structure, this fact is explained by the basic Modigliani-Miller model. Afterwards the authors completed the model taking into account corporate taxation on profits [Modigliani, Miller, 1963]. F. Modigliani and M. Miller defined the impact of tax shield on the firm value; however, they had not a possibility to propose accurate model to be used on practice. The use of the extended model of Modigliani-Miller gave the paradoxical conclusion that the capital structure is optimal when approaching the magnitude of financial leverage to infinity.

For example, there are two companies - A and B. A is fully financed with equity and does not have any debt, and B is financed both with equity and debt. If an investor does not want to

take additional risk, he or she would prefer shares of the company A as it is not liable anything to the creditors. Imagine, the investor bought 1% of the A's shares outstanding. That means that the shareholder has rights for 1% of the company's profits. If the shareholder wants to buy the same share of both equity and debt in the company B, that means that he or she invested 1% in B's equity and 1% in B's debt and as a return he or she will get 1% of the debt interest and 1% of the profit after interest payments. This means that in the end this person will get the same 1% of the company's profit. According to the law of one price, in conditions of perfect market two investments having the same return must have the same price [Brealy, Myers, 2008], so the value of unlevered company A will be equal to the value of levered B.

However, this model does not take into consideration the opportunity to reduce tax payment for the firm B by the amount of debt rate (because debt interest is paid before taxes). This effect is called *tax shield*. As mentioned above, it is a great advantage of the borrowed capital.

In 1963, Modigliani and Miller published a second work dedicated to the capital structure, which entered into the original model of corporate taxes. Taking into account corporate profits taxation, it was shown that the share price is directly related to the use of debt financing: the higher the proportion of borrowed capital, the higher the share price. According to the revised theory of Modigliani-Miller, businesses should be funded only with borrowed capital, as it provides the highest stock prices.

Modigliani and Miller made two propositions related to the relationship between value of levered and unlevered firm, and to the relationship between required rate of return on equity and capital structure. According to Modigliani and Miller, value of levered firm (V_L) is equal to the value of unlevered firm (V_U) adding the gain from the tax shield effect (product of the corporate tax rate r_{tax}, and value of debt D) as shown in the formula 1.3:

$$V_l = V_u + r_{tax} D, (1.3)$$

According to the other conclusion made in the paper required rate of return on equity of the levered firm (r_L) is composed with the return on equity of unlevered form (r_u) and a certain kind of risk premium for the debt presence in the company corrected to the tax shield positive impact (formula 1.4):

$$R_L = R_U + (1 - R_{tax})(R_U - R_d)\frac{D}{E}, (1.4),$$

Where

R_L – required rate of return on equity of the levered firm,

R_U – required rate of return on equity of the unlevered firm,

R_{tax} – corporate tax rate,

D – value of debt,

E –value of equity,

R_D – Rate of return of debt,

E – is equity of the firm.

However, Modigliani-Miller's model has a little implication on real business, because of unrealistic assumptions made: absence of transaction costs, tax rate does not depend on the size of debt and is fixed forever, the debt is also permanent, individuals and corporations borrow at the same rate.

Various research have tried to modify the theory of Modigliani-Miller who, in order to explain the actual situation, neglected many of the original terms of the theory. It was found that some of the conditions have no significant effect on the result. However, with the introduction of a model of such a factor, as additional financial costs due to poor capital structure, the picture changes dramatically. For example, economy on the tax payments enhances the value of the enterprise with increasing share of borrowed capital, but at some point the value of the company starts to decrease with the further increase in debt capital, as savings on tax payments are offset by rising costs on a riskier capital structure.

Hamada's Equation and its modifications

The first part of the paper has the form of the search for an optimal debt ratio of the company's capital. This search is substantially based on the Hamada's Equation [Hamada, 1972]. R. Hamada proposed the following formula for calculating β_L by combination of the Modigliani-Miller model with CAMP and taking taxes into account:

$$\beta_L = \beta_U [1 + (1 - T)\varphi], (1.5)$$

where:

β_L- Beta levered,

β_U – Beta unlevered,

T- Corporate tax rate,

φ - Debt to equity ratio.

In consequence of quantity of CAMP and Modigliani-Miller models assumptions the equation, worked out by R. Hamada was acknowledged advantageous, but too theoretical. Due to restrictions originally present in the model its implementation to the real companies can lead to serious errors in the results, which was confirmed by various studies. It caused several attempts

of upgrading Hamada's Equation usability for real-world examples by the reduction of the restrictions' quantity.

Particularly, survey figured that using the original Hamada's formula leads to the situation when EBIT of the company varies with the level of leverage. Regarding this problem, Conine developed the following modified formula by adding «the beta of debt» almost immediately after the release of the original article:

$$\beta_L = \beta_U [1 + (1-T)\varphi] - \beta_{DEBT}(1-T)\varphi, (1.6),$$

$$\beta_{debt} = \frac{(R_D - R_F)}{R_{PM}}, (1.7)$$

where:

β_L- Beta levered,

β_U – Beta unlevered,

T- Corporate tax rate,

φ - Debt to equity ratio,

R_D- Rate of return of debt,

R_{PM} – Market risk premium,

R_F – Risk free rate.

Conine's modified formula has the «beta of debt» as a conceptual foundation while this concept has been criticized in several researches [Conine, 1985]. The important drawback of this model is coming from the impossibility of offering such a WACC or VL that would fit the definition of an optimal capital structure. Another problem is attributed to noncompliance with the financial principle, according to which the company's value should fall on the some level of leverage as the efficiency of the tax shield sooner or later will be lower than the required rate of return of the amount of risk On the contrary, the value of the company with increasing leverage grows infinitely as specified in Conine's model.

In an effort to handle described noncompliance R. D. Cohen proposed his own modification of Hamada's Equation [Cohen, 2007]. The central message of this modification is reprising the debt with the following formula:

$$D^{¿} = \frac{(R_D * D)}{R_F}, (1.7)$$

where:

D* – Adjusted debt,

R_D – Return on debt,

D – Debt value of the firm,

R_F – Risk free rate.

Alternatively to previous formula the approach of Conine can be used as a part of Hamada's Equation for finding the optimal capital structure, which is expressed by the sequent set of formulas:

$$V_U^{¿} = E + D^{¿} \times (1 - T), (1.8)$$

$$\beta_L = \beta_u [1 + (1 - T) \varphi^{¿}], (1.9)$$

$$\varphi^{¿} \equiv \frac{\frac{R_D}{R_F} * D}{E}, (1.10)$$

where:

V_U* - Value of unlevered firm,

E – Value of equity,

D* – Adjusted debt,

T – Corporate tax rate,

β_L– Beta levered,

β_U – Beta unlevered,

$\varphi*$ – Adjusted leverage,

R_D- Rate of return of debt,

R_F – Risk free rate,

D – Debt value of the firm,

E – Value of equity.

This type of modification seems to be one of the most relevant in the terms of its appliance to the real companies. Such method of Hamada's Equation usability elevation lets to gain better results and improve the accuracy of the model.

Hamada's equation modifications or some pieces of them can be used for the purposes of calculation of the optimal ratio between equity and borrowed capital of the company. Significance of this idea is proved out by the fact that further attempts of modifying the Hamada's formula are undertaken by researchers even today. For example, modification of the formula, proposed by Conine, forms the basis for another model of Munshi [Munshi, 2014], who avoids the restriction about the company's possibility of borrowing at a risk-free rate and takes

into account the increase in interest rates on the debt, depending on the ratio of debt and equity by empirically determined formula $\alpha = 1 + D$, where α is added to the R_D.

Consequently, it is possible to state the necessity occurrence of existing methodology modification due to huge number of assumptions that should be made to implement the model while most of them are not holding in the real company cases. Since this implementing some propositions for the optimal debt structure identification model, based on Hamada's Equation modifications or some other branches of examined field of expertise is one of objectives of this master thesis, which is clear justification of the relevance of the each reviewed paper for my research.

Trade-off theory

Thade-off theory also states that for each company there is a target capital structure that may be obtained. According to this model, optimal capital structure may be found by analyzing costs and gain of every additional dollar in debt. The gain may occur because of the tax deductibility or increasing free cash flow. The debt costs are related to the probability of bankruptcy and potential agency conflicts between equity holders and debt-holders. In the situation when the capital structure (or financial leverage) is optimal, the last dollar's gains offset costs. As stated in the theory, the same things happen to the firm's dividends. Companies select such an amount of dividends that maximizes its value.

Pecking order theory

Pecking order theory suggests that there is not target capital structure and financial managers choose capital sources in the order: internal capital, borrowed capital and equity as "the last resort" [Myers, Majluf, 1984]. That happens because of the information asymmetry occurring between firm's management and its potential investors, that is why, according to the theory, companies first prefer use of the internal capital which do not imply costs related to the information asymmetry than the short-term debt. After the internal capital in the firm is depleted, it would apply for short-term debt rather than longer-term, and long-term debt to the new shares issue. Myers and Majluf stated that the information asymmetry may be overcame if companies use their plowback instead of issuing new shares.

In other words, retained earnings, they do not provide the problem of the adverse selection that occurs because of information asymmetry. On the other hand, equity is dependent on the problem of adverse selection, whereas debt is a subject of an insignificant impact of such a problem. From the point of view of external investor, equity is riskier than debt, because both have adverse selection problem, but the risk premium is higher in the case of equity. That is why external investor require higher return on equity capital, rather than on borrowed capital. Fama

and French empirically proved that less levered firms are more profitable, according the pecking order theory [Fama, French, 2002].

According to this theory, company will not issue equity being in normal conditions, and in the need of financing deficits, the debt will be issued.

Summing up, despite the fact that classical optimal structure models are widely used and discussed in academic papers, they often require unrealistic assumptions that make them not practically valuable. Moreover, the models based on Modigliani-Miller's theory do not fully and accurately describe the relationship between required return on equity capital and risk of default, because in the situation of company's default even if shareholders require incredibly high return, they will not get it eventually. However, this thesis is aimed at building an applicable model that may be analyzed and leads to the certain practical implications. That is why this paper is focused on the other type of optimal capital structure models defining equity as an option. This type of models, first, seems to be more realistic that enable us to make practical conclusions.

1.2.2. Models based on approach to equity as a real option

Besides classical models using weighted average cost of capital and method of discounted cash flows, there is the second type of models predicting the best capital structure using Black–Scholes–Merton differential equation.

Merton model

In 1973, Black and Scholes [Black, Scholes, 1973] and afterwards in 1974 Merton proposed simple model [Merton, 1974], that related credit risk to the firm's capital structure. Firstly, Black-Sholes model was used as an instrument for options evaluation, but namely R. Merton first applied options theory to the problem of debt evaluation in a case of default possibility.

As known from Finance, value of assets is equal to the sum of values of debt and equity. Furthermore, it is not a secret that debt-holders first satisfy their interests in a company in the case of default, and only thereafter stockholders receive their money. In this way, shareholders' capital is the residual value of firm. Consequently, the value of shareholders' capital may be negative if debt value exceeds assets value. If the value of shareholders' capital is under zero, shareholders may take shares off their hands without any costs.

In other words, shareholders may use do not exercise call-option and leave the firm to the creditors. Taking into account that the value of assets is less that the value of debt, the creditors requirements will not be satisfied fully that means the company's default. If the firm value exceeds the value of debt, its shareholders a sort of redeem the firm for the amount of money equal to the company's debt. Otherwise, they leave the firm and the "redeem agreement" is not realized that means call option is not exercised.

Consequently, assuming the assets value to be independent of firm's capital structure, the equity value may be explained as a call option on the company's assets:

$$Call\,option_{short} = D_{risk} - V_{firm}, (1.11)$$

Where

Call $option_{short}$ – price of the call option on the company's assets in short position.

D_{risk} – Value of the firm's debt,

V_{firm} – Value of the firm.

Or in terms of Black-Sholes model:

$$E_t = V_t N(d_1) - e^{-r(T-t)} F N(d_2), (1.12)$$

Where

E_t – equity value at the time t,

V_t – value of the company's assets at the time t,

$N(d_1)$ and $N(d_2)$ – standard normal distribution functions,

T – maturity of the debt,

F – exercise price of the option.

So, if the value of assets (Vt) is higher than the value of debt (F) at the maturity (T), debt-holders will receive F and shareholders V_T - F. In the opposite situation when the face value of debt is higher than the assets value, the company announces default and control over it moves from shareholders to bondholders. This model assume that default will occur at the maturity, where $N(-d_2)$ is the risk-neutral probability of default.

Black-Cox model

In 1976 F. Black and J. Cox proposed the model [Black, Cox, 1976] which assumes that default may happen at any time before the maturity of the debt. As in the previous case, default occurs when the assets value (value of the company) decreases, but in the Black-Cox model this decrease is not defined with the value of debt, but with the specific level that is not constant.

The authors make an assumption that the company observed made only one issue of bonds that do not imply coupon payments (discount bonds). Black and Cox also assume the trigger level (the level of the firm value when default will occur) to be:

$$C_{1t} = C * e^{-\gamma(T-t)}, (1.12)$$

Where

C - safety covenant at the present moment,

C_{1t} – safety covenant at the moment t,

γ – discount rate,

T – debt maturity.

As the time of exercise is undetermined, the authors suggest the continuous time analysis using exponential form of discount rate.

Then the time of default may be defined as follow:

$$\tau = \infty \left\{ s \geq t \middle| V_S \leq K \right\}, (1.13)$$

Where

τ – time of default,

t – present time,

V_S –value of the firm.

K – value of the covenant C_1.

Then the probability that default occurs before the time τ under conditions of risk-neutrality may be expressed as follow:

$$P[\tau \leq T] = N\left(\frac{lnV - lnK + \left(r - a - \frac{1}{2}\sigma^2\right)(\tau - t)}{\sqrt{\sigma^2(\tau - t)}}\right) - \left(\frac{V}{C_{1t}}\right)^{1-\frac{2(r-a-\gamma)}{\sigma^2}} N\left(\frac{2\ln C_{1t} - lnV - lnK + \left(r - a - \frac{1}{2}\sigma^2\right)}{\sqrt{\sigma^2(\tau - t)}}\right)$$

Where

τ, T, V, K, and C1t have been defined earlier,

N – standard normal distribution functions,

r –interest rate,

a – dividend rate,

σ^2 – instantaneous variance of the return on the firm.

The main difference between Black-Cox and Merton model is that the first one accepts the situation when the default may occur earlier that the debt maturity (option expiration time). In this case, we consider European option.

In order to employ this model, authors make several assumptions:

- "Every individual acts as if he can buy or sell as much of any security as he wishes without affecting the market price;
- There exists a riskless asset paying a known constant interest rate r;
- Individuals may take short positions in any security, including the riskless asset, and receive the proceeds of the sale. Restitution is required for payouts made to securities held short;
- Trading takes place continuously;

- There are no taxes, indivisibilities, bankruptcy costs, transaction costs, or agency costs;
- The value of the firm follows a diffusion process with instantaneous variance proportional to the square of the value" [Black, Cox, 1976];

Leland-Toft model

Leland and Toft proposed an extended variation of Black and Cox model in 1996 [Leland, Toft, 1996] that provided the choice of both maturity of the debt and its amount. According to this model, default is defined mostly by endogenous factors, rather than by some external factors or cash flows problems. Leland-Toft model presents wider range of possible optimal capital structures than Leland's previous article published in 1994 [Leland, 1994]. Furthermore, this model enables to find not only optimal amount of debt, but also optimal maturity of the debt, credit spreads, default rates and reduction in the assets value. It is also stated by the authors that predictive power of the model is close to the reality that was checked on historical data. Moreover, this model make difference between short-term and long-term debt, as the first one do not provide such advantages due to the tax shield effect. That is why, according to Leland and Toft, gain from the debt tax shield must be contrasted by the risk of assets transfer that lie on the bond-holders' shoulders. Leland-Toft model is presented in formulas 1.15-1.17,

$$V_b = \frac{\left(\frac{C}{r} \times \left(\frac{A}{rT} - B\right)\right) - \frac{AP}{rT} - \frac{\tau Cx}{r}}{1 + \alpha x - (1-\alpha) B}, (1.15)$$

Where

$$A = 2ae^{-rT} N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}} n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}} n(a\sigma\sqrt{T}) + z - a, (1.16)$$

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right) N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}} n(z\sigma\sqrt{T}) + \frac{1}{z\sigma^2 T} + z - a, (1.17)$$

$$a = \frac{\left(r - \delta - \frac{\sigma^2}{2}\right)}{\sigma^2}, (1.18)$$

$$z = \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{\frac{1}{2}}}{\sigma^2}, (1.19)$$

$$x = a + z, (1.20)$$

V_b – endogenous bankruptcy trigger, when tax deductibility is lost,

r – risk-free interest rate,

a – Bankruptcy costs,

σ – Assets risk or volatility of the firms' assets,

T – Debt maturity,

N – Standard normal distribution function,

C – Coupon payment,

δ – Payments to shareholders.

The main benefit of the model is that it predicts more accurate and interesting results that may be analyzed and applied on practice. As an improvement of the [Leland, 1994] model Leland and Toft proposed optimal capital structure taking into account debt principal value, but not only coupons (as done before). That seems to be more realistic scenario of default as the main problem that causes the company's bankruptcy is inability to pay of the principal value, but not coupons payments. However, the scope of the master thesis do not allow to use such a complex model requiring many resources. That is why it was decide to use Leland model described in [Leland, 1994], as it is less resources requiring, but also more realistic than other models using equity as an options.

2. METHODOLOGY OF IDENTIFICATION OPTIMAL CAPITAL STRUCTURE

In this chapter, in order to complete fourth and fifth objectives of the thesis, firstly the model developed by Hayne Leland will be described, and after this, the modifications of it will be described. As it was already stated, the model invented by Hayne Leland was chosen, on the one hand due to its' positive sides, and close to reality approach, and in the other hand, because of its simplicity. Of course, for more realistic results it would be better to use modification of this model that have been done by Leland himself in 1994 [Leland, 1994], to include time to maturity and the principal of debt. However, the process of solving that model and having a strict – numbers result seems to be the task that lays far-beyond the master thesis requirements. So, in order to achieve thesis objectives four and five this chapter was divided by two parts: firstly, the base model will be described, and in the seconded part necessary modifications will be described, and the standard algorithm, that should be implemented for the real company in order to obtain results in terms of optimal leverage structure will be described.

2.1. Theoretical model description.

To begin with, it is important to state that corporate capital structure and value of debt are interlinked variables. The debt value, because it depends on yield spreads, also depend of level of leverage, because higher leverage level, means more risk in the company, and higher risk means higher yields. The model developed by Leland in his article is further development of the model of Brennan – Schwartz [Brennan, Schwartz, 1978],that provided one of the first quantitative examination of optimal leverage. In their model Brennan and Schwartz made the assumption that unlevered firm value (the value of business) follows a diffusion process with constant level of volatility. However, in Brennan Schwartz model there were several limitations, that were eliminated in the Lelands' model.

First and most important one is that the model was based on the numerical approach, so, it could not give any general solutions for value of the risky debt and optimal choice of leverage. Of course numerical examples are quite useful, but they cannot be applied generally to the possible companies.

Second one is that the model of Brennan – Schwartz is built around certain case, where the bankruptcy of the company triggers only when the business value falls to the principle value of debt. Nevertheless, the described situation is not really common, or even, really uncommon in the real cases of the companies. The described situation is commonly fined only in the cases of the short-term debt contracts, but is rare in long term debt cases, which was described by the future researches [Smith and Warner, 1979]. In current Masters' thesis, the model is developed

for the long-term debt cases, so the alternative bankruptcy triggering conditions, described in the Leland's model are more suitable.

Third limitation of the Brennan- Schwartz model is the maturity dates, which also are quite short. Even in their model they take long-term debt, with big time to maturity as an example, it was already discussed before, that the conditions implied bi Brennan and Swartz are mostly true for short term debt, so again, for this Master thesis model implemented by Leland is more suitable. All the limitations of the Brennan and Schwartz model were eliminated in the Leland's model, as it will be shown further,

Leland's article considered two possible options of bankruptcy determinants: first is when bankruptcy is triggered endogenously – when firm has insufficient equity capital to meet it current debt obligations. The second when it was triggered similar to the Brannan Schwartz model conditions, and in the Leland article it is called – the case of protected debt. However, due to more general approach, the model of the current Master's thesis will concentrate only on the first case of the Leland's model that is called unprotected debt.

In the Leland's model there is important approximation that corporate securities depend on the underlying firm value, but, are time-independent. The author of the article states, that despite the fact that debt securities in most cases have a specified maturity date and, therefore have time-dependent cash flows and values, the time independence assumption can be justified. In some cases very long time horizons, are as good as infinite, since after a certain moment, the value of the future payments are nearly zero in the present moment. And very long time horizons for fixed obligations are not new, either in theory or in practice. They were used in [Modigliani Miller, 1958] model had assumptions of infinite maturity debt, [Black and Cox, 1976] took infinite maturity debt in their model and in real life as well some examples can be found. For instance, Bank of England issued Consols – bonds with a fixed coupon with infinite maturity, as well as preferred stock pays dividend without any time limit. The other explanation given by the author if following: "time-independent environment is when, at each moment, the debt matures but is rolled over at a fixed interest rate (or fixed premium to a reference risk-free rate) unless terminated because of failure to meet a minimum value, such as a positive net-worth covenant.". Basically, it means, that the debt policy of the company is that they take some debt to cover previous debt, so the leverage level is keeping the same trough time. This policy can be used by the firms, so because of this two explained reasons in this Master's thesis, as well as in the Leland's model the time of debt maturity considered to be infinite and assumption can be considered as realistic.

This time independents assumptions allows to establish closed-form solutions for optimal capital structure, and the Leland's article itself is a further development of the results gained by

following researchers [Merton, 1974], [Black and Cox, 1976] with taxes and bankruptcy costs. The article itself has following research questions to cover:

- How do yields spreads depend on corporate debt, firm leverage, taxes and risk free rate
- What is influenced by the level of leverage (bankruptcy costs, tax shields, etc.)
- How bankruptcy risks depend on level debt and leverage

So, as it can be seen the research questions of the Leland's article is quite similar to research questions of the current Master's thesis. However, in the Leland's article, there is only theoretical description of the model. In my Master's thesis, I will modify the Leland's model to show the optimal leverage level calculation procedure.

Next assumption if the Leland's model is that the face value of debt remains constant in time. However, in the model developed in current Master's thesis that assumption is modified, so only the leverage level should keep constant which seems to be more realistic assumption, since there are quite a lot of companies that have constant leverage policy. Anyway, it seems to be quite realistic assumptions and H. Leland proves it in the last section of his work, when it is shown that issuing additional debt will hurt debt holders, however, this part of work will not be described, since this topic relays beyond the topic of current Master's thesis. All in all, this is general description of the roots of the model and its preresearches, and now it is time to describe the model itself.

Description of a model with time –independent security values

To begin with, it should be stated, that business value – the value of unlevered firm, follows the diffusion process with constant volatility of rate of return and W as a standard Brownian motion:

$$dV_t = \mu(V, t)\,dt + \sigma dW, \quad (2.1)$$

As it can be seen, this value is the value of firm's assets, and does not depend on choice of leverage. This approach to business value is quite common in the financial science, so the author of the current Master's thesis, believes that this assumption about firms unlevered value is quite accurate, and can be applied for the cases of the real companies. Another important assumptions, that any cash outflows connected with debt payments, which depend on the choice of leverage, are financed with issuing additional equity. This assumption also seems quite reasonable, since – firstly it is necessary to fulfill the general theoretic statement, that in the "perfect world" without transaction costs and taxes the value of the firm does not depend on its financial structure, and secondly, as Leland states : "this is consistent with the bond covenants

that restrict firms from selling assets". The same assumption is made on the Brennan and Schwartz model and other research papers, so it seems to be quite reasonable and applicable for the real company's cases. However, this assumption is quite important, when we will test the model on the real companies, since, according to this assumption, the company must take their debt not to finance its' current activities, but to invest in some fixed assets or similar projects. This assumption is one of the reasons, that for the testing companies were chosen huge oil companies, that uses their debt not to finance its current activities (like resellers, for example), but to invest in huge long-term projects.

So, following Modigliani Miller, black and Cox and Merton, it is assumed that some riskless asset exits, and it pays rate r as an interest. To proceed with, it is assumed, that firm constantly pays some non negative coupon – C – every period in time, until default. This coupon is return on debt, which is paid by the firm to the debt holders, presented in some value. Basically this coupon is return on debt, required by debt holders, multiplied on the total value of debt. F(V,t) is value of such a claim, which depends on value of business and time. It was proved by Black and Cox that any assets value, must satisfy this partial differential equation, if firm finances the cost of coupon by issuing additional equity:

$$\left(\frac{1}{2}\right)\sigma^2 V^2 F_{VV}(V,t)+rV F_V(V,t)-rF(V,t)+F_t(V,t)+C=0,(2.2)$$

here, coupon outflows should be paid at maturity, and they should be paid until bankruptcy accrues. In general, closed form solutions cannot be found for this equation. Nevertheless, if we state securities time independence, due to the assumptions made above, $F_t(V,t)$ equals to zero, so this equation becomes ordinary differential equation:

$$\left(\frac{1}{2}\right)\sigma^2 V^2 F_{VV}(V)+rV F_V(V)-rF(V)+C=0,(2.3)$$

And it has a solution:

$$F(V)=A_0+A_1 V+A_2 V^{-2r/\sigma^2},(2.4)$$

And now A_0,A_1 and A_2 can be determined with different boundary conditions. So, it was proved, that all time-independent claims, that satisfies previously made assumptions, have this functional form. After this, it is time to determine specific values, based on this equation in order to answer stated research objectives. Also, to simplify the following formulas X will be used instead of $2r/\sigma^2$ ($X=-2r/\sigma^2$).

AS it was already stated, firm pays constant coupon for the value of debt. This value depends on value of unlevered firm and coupon. So the debt value can be described as a function D(V,C).The coupon, however can be suppressed from this function since it is required rate of return on debt multiplied by debt, and it depends on debt, so the debt can be described as D(V).

Also, it should be considered that the Vb is the level of asset value, when the bankruptcy occurs. We assumed, that if Vb is reaching V, than the firm is declared as bankrupt, since it cannot meet its' debt obligations. This value will be determined later, and for now we consider it as some fixed value with the condition that if V≤Vb the bankruptcy is triggered. What is also important is that, if firm declares bankruptcy, then equity holders are left with nothing, since every asset that are left from the firm are going to the debt holders, due to law restrictions. However, there are some losses of assets, associated with the firms bankruptcy. The value that will be lost is named α, and it's a fraction of the value that is lost and it can change from 0 ≤α≤1. It should be stated, that for every firm that value is unique, but there are some common numbers for all the firms, and according to researches, they are near 0.5 So, in the case of bankruptcy shareholders will gain nothing, and debt holders will gain (1-α)*Vb, or in other words, debt holders will gain all left value of firm (since bankruptcy accrues only when V≤Vb), except bankruptcy costs. Also, the value α can be defined as a recovery rate. So, with fixed (for now) Vb, and knowable α, debt value D(V) can be determent, since debt value is a form of equation F(V), we can found it, identifying A_0,A_1 and A_2. The boundary conditions are following:

$$\text{If } V=Vb,\ D(V) = (1-\alpha)Vb, \qquad (2.5)$$

$$\text{As } V \rightarrow \infty,\ D(V) \rightarrow C/r, \qquad (2.6)$$

This means, that if business value falls to the bankruptcy triggering asset value, then debt cost the amount that will be given to debt holders, and this amount is part of the Vb except cost of bankruptcy. On the other hand, if the business value is really large, then the debt value is the value of all coupons that will be paid by the firm (since, the firm will never collapse in terms of infinite value of business, the value of debt is just a value of infinite annuity, with risk-free rate, since firm with infinite value of business is risk free). From this conditions, and equation F(V), it can be found that in this case, A_1=0 and , since $V^{-X} \rightarrow 0$ as $V \rightarrow \infty$, A_0=C/r. Also, because of first condition $A_2=\left((1-\alpha)Vb-\frac{C}{r}\right)\left(\frac{V}{Vb}\right)^{-X}$. All in all with all A defined< we consume that:

$$D(V)=\frac{C}{r}*\left(1-\left(\frac{V}{Vb}\right)^{-x}\right)+\left(\frac{V}{Vb}\right)^{-x}*((1-\alpha)*Vb), (2.7)$$

or:

$$D(V)=\frac{C}{r}*(1-p)+p*((1-\alpha)*Vb), (2.8)$$

where

$$\frac{V}{Vb}$$
¿
¿
$$p = ¿$$

It can be said that p here is the present value of 1$ contingent, in case the bankruptcy appears. This formula for debt is developed results of Black and Cox model, but with bankruptcy costs. The taxes here change the value Vb, that will be shown later, so it can be said that tax benefits are included in this model. AS it was already mentioned, debt changes the firms total value in two ways: firstly, it cuts the value due to higher riskiness of the company, but it also increases value due to tax shields. Both this effects are time independent, however, they both depends on the value of unlevered firm. So, both of them: tax benefits and bankruptcy costs can be valued as time independent securities, through the formula F(V).

Firstly, bankruptcy costs will be identified. If the company declares bankruptcy – the value V falls to Vb, then bankruptcy costs accrues, and they can be defined, as amount lost to bankruptcy costs- BC in the following work. However, if the firm did not declare bankruptcy, then BC are equal to zero. In other words, if firm will never collapse (as it was explained before, it would happen when business value of the firm is infinitely large), the bankruptcy costs will never occur. This gives the following conditions:

$$\text{If } V=Vb,\ BC(V) = \alpha Vb\ , \qquad (2.10)$$

$$\text{As } V \rightarrow \infty,\ BC(V) \rightarrow 0\ , \qquad (2.11)$$

So, for this conditions, considering equation F(V), bankruptcy costs can be defined as:

$$BC(V) = \alpha * Vb \left(\frac{V}{Vb}\right)^{-X}, (2.12)$$

Or:

$$BC(V) = \alpha * Vb * p, (2.13)$$

What basically means that, present value of bankruptcy costs is the amount lost due to bankruptcy multiplied on the present value of 1$, in case if bankruptcy occurs, and this results corresponds to common sense. Of course, bankruptcy costs are decreasing function of V, since more business value the firm has, less likely it will go bankrupt, and bankruptcy costs will occur.

The value of tax benefits occurs with debt financing can also be defined through equation F(V). Firstly, it is assumed that tax rate that company pays is considered known and have value of τ. This tax is paid until the bankruptcy is declared and is time-independent. The value of tax benefits – TB in the following work - is the same in all periods and depends on the amount of coupon paid on debt (which is known and constant for all periods) and equals tax rate multiplied by coupon value – τ*C. However, in case of bankruptcy tax benefits are lost and equals to zero.

In case of the firm that will not declare bankruptcy at any moment (risk-free firm, with value of business near to infinite) the tax shields will appear at all periods. So from the declared above, boundary conditions are following:

If V=Vb, TB(V) = 0, , (2.14)

As V → ∞, TB(V) → τC/r, , (2.15)

So, solving F(V) for TB(V) under above condition gives:

$$TB(V)=\frac{\tau*C}{r}-\frac{\tau*C}{r}*\left(\frac{V}{Vb}\right)^{-X},(2.16)$$

This formula indicates the value of tax benefits, considering the risk of bankruptcy. Of course, this is rising function of V, since more business value firm has, less likely it will go bankrupt. However, this approach to tax benefits requires an important assumption that firm will fully benefit from tax shields at all time but it is not always true. Firstly, to have this benefits firm must have EBIT higher than zero. Moreover, under some tax codes (U.S. tax code, for example), to benefit fully from the tax deduction, firm must have EBIT at least as large as coupon payments. So, it should be taken into account and the model should be tested only on those corporations, for which this assumptions are true.

Now it's time to find the overall firms value. Due to the logic of the model, the total firms value consists of three components: value of unlevered firm, or business value, benefits gained from the taxation, due to choice of leverage and bankruptcy costs. Of course, tax benefits and bankruptcy costs are calculated with the respect of bankruptcy probability, and higher is this probability, higher are bankruptcy costs and smaller tax benefits. Bankruptcy probability is considered in Vb, and later in this work it will be shown how, but for now regards it as fixed. So all in all, total value of the firm can be presented as:

$$u(V)=V+TB(V)-BC(V)=V+\left(\frac{\tau C}{r}\right)\left(1-\left(\frac{V}{Vb}\right)^{-x}\right)-aVb\left(\frac{V}{Vb}\right)^{-x},(2.17)$$

Here it can be seen that u(V) is rising function of V, and it make sense, since more value in business there is, more valuable is a company itself. However, it should be take into account, that this equation can be applied only with some logically based assumptions, that coupon is non negative, and there are some taxes and recovery rate that are higher than zero, but less than one. Moreover, it is interesting to mention that in case V=Vb, u(V)= (1-α)Vb, which also corresponds to the common sense, since if the value of the firm falls to Vb (the bankruptcy is triggered) , then all the company is worst is the amount of assets (except sunk costs) that are left for debt holders.

The value of firms equity can be presented as:

$$E(V)=u(V)-D(V)=V-\frac{(1-\tau)C}{r}+\left(\frac{(1-\tau)C}{r}-Vb\right)*\left(\frac{V}{Vb}\right)^{-x},(2.18)$$

This equation was gained with the simple algebra. Here it also can be seen that E(V) is rising function of V, since – more business value firm holds, more its equity worth. What is more, it should be stated that according to the previous assumptions, as well as real world situations, E(Vb)= 0, due to the fact that in case of bankruptcy equity holders gain nothing from the company's leftovers. This reflects the "option-like" nature of equity, with Vb as a strike price. However, this approach has some possible agency problems associated with the "asset substitution". This is a problem, when equity holders become risk neutral, from some point of leverage, since they will gain nothing in the case of bankruptcy anyway, so they chose the most risky projects in order to gain higher returns. This is an interesting question to discuss, however, this problem stands beyond the goal and objectives of this master thesis, so in the current work, it will be assumed that this problem is not really going to take place. Of course, this assumption limits our testing companies in such a way, that the author of the thesis has to be quite sure, that there is no this problem taking place in the researched companies.

As it was shown Vb is a crucial parameter for the model. Vb is the level of asset, which triggers bankruptcy, since company has not enough recourses to meet its obligation, and when it happens, equity value of the company is falling to zero, and debt value to Vb without a fraction lost due to the bankruptcy costs. In order to calculate this amount several important assumptions should be made. Firstly, it is understandable that u(V) will be maximized with the Vb setting as low as possible, under other fixed parameters. However, it is stopped by being smaller than it possibly can be due to limited value of equity, since due to previously made assumption – firm finances its coupon payment by equity. So, the equity value E(V) must be nonnegative for all V>Vb and eq1uels zero for V=Vb. So, we assume that Vb is the smallest possible value, for the given E and other parameters. Since, it was proven that E(V), under common environment (taxes and recovery rate from zero to one, coupon non-negative) is rising function in terms of V, the lowest possible value of Vb is when differenced E(V) = 0 , or dE/dV = 0. Moreover, due to "option-like" nature of equity its value is equal to zero, when V=Vb. So at the same point, lowest equity value is zero (due to common sense) and it also happens at the point when V=Vb Thus to calculate Vb, we should fulfill all this requirements. The dE/dV will have the following form:

$$\frac{dE}{dV}=1-\left(\frac{x*\left(\frac{V}{Vb}\right)^{-x-1}*\frac{(1-\tau)*C}{r}-Vb}{Vb}\right),(2.19)$$

Thus, now showld be defined a value when dE/dV = 0, and at the same tome V=Vb, so:

$$0=1-\left(\frac{x*\left(\frac{V}{Vb}\right)^{-x-1}*\frac{(1-\tau)*C}{r}-Vb}{Vb}\right),(2.20)$$

Or:

$$0=1-\left(\frac{x*\left(\frac{Vb}{Vb}\right)^{-x-1}*\frac{(1-\tau)*C}{r}-Vb}{Vb}\right),(2.21)$$

Solving this in terms of Vb gives:

$$Vb=\left(\frac{(1-\tau)C}{r}\right)\left(\frac{X}{1+X}\right),(2.22)$$

Or:

$$Vb=\frac{(1-\tau)C}{r+0.5*\sigma^2},(2.23)$$

Now Vb is defined. As it can be seen, Vb is proportional to coupon. It happens, because the higher level of leverage the company has, the higher risk it takes, and higher return on debt (coupon in this model) it has to pay. Also, Vb decreases with increase in tax rates, since more tax it is required to pay by the firm, more tax shields it gains, so more valuable becomes the debt. Also, the Vb decreases as the risk free rate rises. It corresponds to the common sense, since the the risk of the firm is proportional to the overall risk at the market, and more valuable becomes risk-free activities, more valuable becomes equity of the firm. It is also interesting, that the Vb is independent of the value of unlevered firm. This situation occurs, since the influence value of business on Vb is already counted in the model by the parameter p, so if the V would influence the value of Vb, then the results would be incorrect, since “double counting” will appear. However, as it was already stated, in this model the only risk that is counted in this model, is the internal risk of default. But in the real world there are plenty more risks that appears in the company. In this model, author of the current master thesis decided to include currency risk in this model. Of course, even with currency risk the model will steel be quite an approximation it terms of the risk analysis, but it still be quite relevant if used properly.

So, all in all, this is the theoretical model for unprotected debt. The comparative statistic of it is presented below.

Variable	Homogeneity	Shape	Limit As		Sign of Change in Instrument for an Increase in:					
			$V \to \infty$	$V \to V_B$	C	σ^2	r	α	τ	V
D	Degree 1 in V, C	Concave in V, C	C/r	$\frac{C(1-\alpha)(1-\tau)}{(r+0.5\sigma^2)}$	> 0; $< 0^*$ as $V \to V_B$	< 0; $> 0^*$ as $V \to V_B$	< 0; $> 0^*$ as $V \to V_B$	< 0	> 0	> 0
R	Degree 0 in V, C	Convex in V/C	r	$\frac{(r+0.5\sigma^2)}{(1-\alpha)(1-\tau)}$	> 0	> 0; $< 0^*$ as $V \to V_B$	> 0; $< 0^*$ as $V \to V_B$	> 0	< 0	< 0
$R - r$	Degree 0 in V, C	Convex in V/C	0	$[0.5\sigma^2 + r(\alpha + \tau - \alpha\tau)]/[(1-\alpha)(1-\tau)]$	> 0	> 0; $< 0^*$ as $V \to V_B$	< 0	> 0	< 0	< 0
v	Degree 1 in V, C	Concave in V, C	$V + \tau C/r$	$\frac{C(1-\alpha)(1-\tau)}{(r+0.5\sigma^2)}$	> 0; $< 0^*$ as $V \to V_B$	< 0; $> 0^*$ as $V \to V_B$	< 0; $> 0^*$ as $V \to V_B$	< 0	> 0	> 0
E	Degree 1 in V, C	Convex in V, C	$V - (1-\tau)C/r$	0	< 0	> 0	> 0	0	> 0	> 0

Figure 2.1.1 Comparative statistics of financial variables

In this table, there is the description, of how variables influence each other. Here, R is return on debt, or C/D(V). And R-r is basically a yield-spreads required on debt. This is theoretical analysis, and any firm that have this theoretical model, will act in the following way. So, now it is time to modify a model, firstly to add currency risk and define an optimal level of leverage.

2.2 Model development

In order to achieve fourth and fifth objectives of the research, the model of Leland should be modified in way, that will allow to identify the optimal capital structure of the real company. Before, making any modification to the theoretical model, there should be considered several assumptions that should be taken into account, so the model would give correct results:

The assumptions of the model are following:

- The firms value of the assets, Vt , can be described by a diffusion-type process with stochastic differential equation $dVt = m(V,t)dt + \sigma dW$
- The stochastic process of V is unaffected by the financial structure of the firm
- All outflows associated with the choice of leverage are financed by selling additional equity and Debt of the firm is unprotected
- The cost of debt can be described as perpetual coupon that is paid to the debt holders once a year, until default
- If firm keeps constant D/E ratio , the C/D(V) ratio also remain constant
- Rate of recovery is unaffected by the financial structure of the firm
- Considering a constant corporate tax rate , the firm obtains tax shields from its debt at a rate C until default
- Bankruptcy occurs when the firm value reaches a threshold Vb, and firm is trying to keep Vb as low, as possible at certain level of leverage

Now, if the assumptions holds, the parameters of the model can be estimated. It is important to state that al, the parameters and proxies are found for the certain time moment. In this master's thesis, there were four time moments taken : first January 2010, first January 2013, first January 2014, first January 2015. This particular time moment were chosen, because author of the thesis decided that it would be interesting to test the model in different economic situations. As it is known, 2014 and 2015 were crisis years for Russia, and especially large oil companies, because of two factors: low prices on oil, and huge fall of rubble currency. The first factor is important due to quite obvious situation, that low oil prices damage the oil companies. The second factor, however, is important because large Russian companies have a huge debt in Eurobonds. So, combination of this two factors damaged this type of companies quite a lot, and that's why those years were taken. I chose 2010 and 2013 for camperecent pre-crisis years, so the difference in

optimal leverage structure suggested by the developed model will be seen clearly. However, the algorithm used is the same, despite the time moment taken. Moreover, in every moment in time there is an assumption that main proxies will stay the same for infinite time, for simplicity and accurateness of the model in terms of mathematics. In the future studies, and model development, it is recommended to change the model, so it can show the time-dependence and change with the changing environment, but this task is far beyond the field of master thesis competence, so for now the model will be calculated as it was described previously.

The parameters that are proxied are following:

- Risk free rate r;
- Tax rate τ;
- Total firm value u(V) ;
- Equity value E(V);
- Debt value D(V) ;
- Monthly volatility of equity returns $\sigma_{m;}$
- Monthly average of equity returns μ_m .

The proxies were chosen with respect to the previous studies, and common sense.

Risk-free rate of return reflects the level of income that could be received by investors without incurring the risks associated with the investment. It is believed that the risk free rate is the same for all investors, at least in one country and currency. However, there is no absolutely no risk object of investment, so as the risk-free rate is usually is used the yield on government securities of the country, in which the company operates. Debt securities are usually issued to certain terms. Because the model is mainly conducted on the basis of the infinite operation of the company (or until bankruptcy) as the risk-free rate should be selected state bonds with the largest remaining term to maturity. Due to this reasons, as a risk free rate the long term rate on the Federal Loan Obligations (OFZ) in the certain date were taken. Those are the state Russian bonds, issued by government of Russia and we take the longest maturity period possible.

Tax rate influences the all the model equations quite a lot, since, the tax benefits dependent on the tax rates as well as the level on assets that triggers bankruptcy. The problem here, is that the companies that are analyzed and on which the model is tested has earnings not only in Russia, but abroad as well. So, in order to simplify the model, and eliminate the need to reflect all the possible taxes and different tax rates in tax codes of different countries it was chosen to use an effective tax rate. This rate is calculated by dividing tax paid, taken from the balance sheet, on profit before tax. This approach was used by many researchers [Patel and Pereira, 2005], who were testing similar models, and the results that were gained in their works were quite reasonable, so the same approach will be used in the current master's thesis.

As a total firm value at a certain point in time market capitalization was taken. This is also a common approach [Ericsson and Reneby, 1998], and seems to reflect the real situation quite well. The market capitalization of the company - a financial indicator, which determines the market value of the company, based on the current value of the shares on the stock market. Calculate the company's market capitalization by multiplying the number of shares issued in circulation at their cost, the current stock market. In this particular case, in order to have more unbiased results, as a share price is taken the average share price through the year before the analyzed date. Foe example for 01.01.2010, the price for the share will be average price from 01.01.2009 to 01.01.2010. This allows reflecting average company value trough a year, when it is analyzed.

As a proxy for the debt value, the balance value of debt was taken. The balance value of debt is short term debt plus long term debt. The debt was adjusted in the way that most of the non-currency debt was subtracted from the debt value. However, in terms of simplicity and more analytical model testing all left debt was assumed to be euro-bond debt. So, to have more accurate results, the companies that were taken, have huge value of its debt in Eurobonds. The approach to debt value as a balance debt value, was already used in the previous studies, that were testing the similar models [Teixeira, 2007]. So, this approach can be called unbiased and accurate.

Since value of the firm is sum of debt value and equity value, the value of equity for a certain time period was calculated as E(V)= u(V)- D(V). Because u(V) and D(V) was described previously, we consider equity value of the firm as a known parameter.

Now it is needed find the volatility of the firm's equity. It is needed to find the volatility of unlevered firm, with the help of Ito's lemma. We make an assumption, that the volatility of early returns of the total firm is the same as volatility of equity returns of the firm. This can be proven by the fact that as it was mentioned above, equity is a simple linear function of the total firm value, since E(V)= u(V)-D(V), and debt remains constant. The volatility of the u(V) can be described as volatility of firm's shares. However, this assumption also makes a boarder to the number of companies that can be analyzed with proposed model, because the shares of the analyzed firms should have high liquidity. Another problem associated with the volatility of firms equity is that it should correspond to chosen time periods- in our cases its 1 year. But the year volatility for certain year cannot be found straight from the share prices. So, in order to find it, we will calculate it on the basis of monthly volatility for the analyzed year and the monthly average for the same time period. So, as a proxies for this numbers the monthly volatility and monthly average of the share prices returns will be taken, with the respect to the time period –

for example, for the date 01.01.2010 the will be taken monthly numbers from 01.01.2009 to 01.01.2010.

2.2.1. Model parameters estimation

Now, when all endogenously determined proxies were explained, the estimated parameters of the model can be analyzed. The process of estimating the parameters of the model will consist of the two steps. Firstly, the parameters without any currency risk would be estimated. Secondly, this parameters will be adjusted in order to include the currency risk of debt. This approach is required due to technical limitations, since most of the equations in the model cannot be solved analytically, and requires numerical solving in order to gain results.

First estimated parameter will be yearly volatility of the returns of equity. This parameter will be the same, regarding of the currency risk, since the volatility of the company's equity is determined with the help of company shares volatility, where it is believed, the currency risk associated with the company is already included. The early volatility of the company's equity returns can be found from the following equation:

$$\sigma_E^2 = \left(\sigma^2{}_m + \left(1+\mu_m\right)^2\right)^{12} - \left(1+\mu m\right)^{24}, (2.24)$$

Where, σ_E^2 is squared yearly volatility of company's equity. Since, all the parameters in this equation are already defined, we simply compute yearly equity returns volatility.

It is important to state that, as it was described before, the model uses very specific approach to the debt of the company, as a perpetual coupon. It is known, that in real cases, this scenario is hardly ever a case. So, in order to implement the model, the first thing that should be done, is adjusting the existing debt of the company, to the "theoretical" debt, that is used in the model. It can be done by calculating the theoretical coupon, that the firm should pay for the existing amount of debt. Since, we have the today's value of debt – D(V)- it can be used to define the coupon, that is used in the model. However, the theoretical model does not include the currency risk of debt, so the identified coupon should be adjusted on some amount to be more accurate interpretation of the companies required return on debt. So, basically the algorithm of identifying the parametes will be: calculating the coupon without currency risk, then adjusting it to the currency risk, and then, identifying all the other model parameters.

To complete the first step of the model parameters identification process, there is a need to identify following parameters:

- V_{naive} – value of business without currency risk
- σ_{naive} - volatility of business without currency risk
- C_{naive} - perpetual coupon without currency risk

- α_{naive} – rate of recovery without currency risk

Now, to gain a number of equations that can be solved simultaneously, one of this parameters should be specified. In this case, we assume that we will identify the parameter α_{naive}, because of the two reasons. Firstly, the change of this parameter in all possible ways – from 0 to 1, influence model the least, unlike all the other parameters. Basically, it only changes the value of debt, and as it was found by the author of the thesis, the difference between two critical values of this parameter – zero and one, only gives 10% difference in terms of value of debt. In other words $D(\alpha=0) \approx 0.9 * D(\alpha=1)$, so as it can be seen, there is no huge difference. Second reason, is that in most of the works[7], where similar models are analyzed[9], α is most likely to be determined as 0.5. What is more, the studies concerning the recovery rate often give the same results. Several studies in the literature report that, bondholders' recovery rate varies with the type of debt. For example, [Altman, 1991] finds that, during the period 1985-1991, the average recovery rate for a sample of defaulted bond issues was: 0.605 for secured debt, 0.523 for senior debt, 0.307 for senior subordinated debt, 0.28 for cash-pay subordinated debt, 0.195 for non cash-pay subordinated debt. Given this evidence, previous studies: [(Longstaff and Schwartz, 1995), (Delianedis and Geske, 1999), (Leland, 2002), (Huang and Huang, 2003)] assume an average recovery rate of 51.31%. So the assumption of recovery rate being 50% is quite reasonable, and, what is more, this value still will be modified into the more precise value in the second step of the models parameters estimation.

In order to gain the third equation of parameters estimation, Ito's lemma should be used. According to the lemma, if one of the processes is a function of the other process, which is the standard Brownian motion process, then the dependent process is also the standard Brownian process, with volatility dependent on the volatility of the second process. In other words, if:

$$dx = \mu dt + \sigma dW, (2.25)$$

And

$$F(x,t) = F(x(t),t), (2.26)$$

Then

$$dF = A(x,t)dt + B(x,t)dW, (2.27)$$

And

$$B^2(x_0,t_0) = \sigma^2{}_0 \left(\frac{dF}{dx_0}\right)^2, (2.28)$$

In our case, value of unlevered firm - V - was defined as a standard Brownian process with constant volatility, and equity was defined as a function of unlevered firm – E(V) – so E(V) is also a standard Brownian process with constant volatility. So, due to this important facts we

can define volatility of unlevered firm as a function of volatility as equity, which was already found earlier. This basically means that:

$$\sigma_E^2 = \left(\frac{dE}{dV}\right)^2 * \sigma^2, (2.29)$$

Now, since values τ, r, σ_E^2 and α_{naive}, as well as E(V_{naive}) and D(V_{naive}¿ were already defined, we have a situation with three equations and three unknowns, which can be presented in the following form:

$$\begin{cases} \sigma_E^2 = \left(1 - \left(x_{naive} \dfrac{\left(\dfrac{V_{naive}}{Vb_{naive}}\right)^{-x_{naive}-1} * \dfrac{(1-\tau)C_{naive}}{r} - Vb_{naive}}{Vb_{naive}}\right)\right)^2 \sigma_{naive}{}^2 \\ D(V_{naive}) = \dfrac{C_{naive}}{r} * \left(1 - \left(\dfrac{V_{naive}}{Vb_{naive}}\right)^{-x_{naive}}\right) + \left(\dfrac{V_{naive}}{Vb_{naive}}\right)^{-x_{naive}} * \left((1-\alpha_{naive}) * Vb_{naive}\right) \\ E(V_{naive}) = V_{naive} - \dfrac{(1-\tau)C_{naive}}{r} + \left(\dfrac{(1-\tau)C_{naive}}{r} - Vb_{naive}\right) * \left(\dfrac{V_{naive}}{Vb_{naive}}\right)^{-x_{naive}} \end{cases}, (2.30)$$

Where:

$$Vb_{naive} = \frac{(1-\tau)C_{naive}}{r + 0.5 * \sigma_{naive}{}^2}, (2.31)$$

$$x_{naive} = \frac{2r}{\sigma_{naive}{}^2}, (2.32)$$

So, now we can define C_{naive}. When it is done, we can proceed to the next step of identifying model parameters. For proper identification of the model parameters the currency risk should be included in the model. To achieve it, we should make changes to the cost of debt of the companies. However, it should be stated, what fraction of the coupon should be adjusted. Since, the large Russian oil companies gain some profits in other currency then rubble, it should be stated, that some part of the currency risk of debt is covered with this profits. In order of simplicity, it will be assumed, that since the debt of the companies is in dollars, all non-rubble profits is also in dollars. This assumption is quite reasonable, since companies themselves state that most of their not rubble profits are dollar profits [Lukoil annual report, 2015]. The other connected assumption will be that the company covers the dollar debt by dollar profits on the same fraction as its dollar to rubble profits. From some point of view this assumption might not hold on practice, since the company is covering all its currency debts with currency profits, however, in this case, the same amount of rubble profit is lost. So, we will say, that due to

simplicity this assumption holds, and the company covers only part of its currency debt with currency profits. This fraction will be defined as:

$$Q=\frac{Net\ Income \in RUB}{(Net\ Income \in RUB + Net\ Income \in USD)},(2.33)$$

Now, we will include currency risk in the coupon. For doing so, we assume, that every year company should hedge the currency risk obligatory. For doing so, the company pays certain amount of coupon, for cover the risk of significant change of the coupon payments, due to currency differences. As a significant amount 2,5% of the coupon payments will be taken. So, basically, the company guarantees, that on the next years, the amount of the coupon payment will not rise more than 102.5% from the coupon. But, as it was mentioned, some of the currency risk is already covered by dollar profits. So, basically, the changeable part of the coupon can grow more than 2.5%, the real amount of growth of can be calculated by the estimation:

$$K=\frac{1.025-(1-Q)}{Q},(2.34)$$

To guarantee that, the company buys a call option, with the strike price

$$Strike=C_{naive}*K,(2.35)$$

The time to maturity of this option is one year. The current price of underlining asset is C_{naive}. One more important assumptions associated with this type of including "costs of currency risk" is that the yearly volatility of the returns of the RUB/ USD pair will be the same for all following years. Of course, this type of assumption is not really the case in the real market situation, but from the point of view of the company from the certain year, it makes sense. However, this assumption was taken in order to simplify the model, and as further model development, I would recommend to revise it. Still, from the theoretical point of view, it has its use, so it will be kept like this. The yearly volatility of RUB/USD returns, was taken from the monthly volatility and average, by the same equation as was described before for the equity volatility returns. Now, the price of this "theoretical" call options is calculated, with the use of Black-Scholes option pricing model:

$$call(C_{naive},t)=C_{naive}*N(d1)-Strike*e^{-r*t}*N(d2),(2.36)$$

$$d1=\frac{\ln\left(\frac{C_{naive}}{Strike}\right)+\left(r+\frac{\sigma_{usd}^{2}}{2}\right)t}{\sigma_{usd}*t},(2.36)$$

$$d2=d1-\sigma_{usd}*t,(2.37)$$

Where:

- C_{naive} — the value of estimated coupon

- N(d) — normal distribution function
- Strike — strike price of option;
- r — risk free rate
- t — time in years, in this case its one year
- σ_{usd} – early volatility of RUB/USD returns.

After defining the price of an option, we add it to the coupon. This represented the currency risk in a fact, that for more risky debt (due to higher currency risk), firm has to pay higher return on debt. Also, as well as the coupon, firm has to finance this additional spending with issuing more equity. What is more, it is assumed, that if firm is unable to pay this additional price of debt, it declares default. This assumption is true, since the call option here represents not the real hedging case, but the additional costs of currency risk, so it means, that if firm cannot afford to do hedging, the required costs of debt had risen too high due to currency change, and now firm is unable to meet its obligations. So now the perpetual coupon is:

$$C = C_{naive} + call\left(C_{naive}, 1\right), (2.38)$$

After identifying the coupon with included currency risk, the other parameters of the model should be estimated. Again, with the help of Ito's Lemma and identified τ, r, σ_E^2 and C, as well as E(V) and D(V) the system of equations can be build:

$$\begin{cases} \sigma_E^2 = \left(1 - \left(x \dfrac{\left(\dfrac{V}{Vb}\right)^{-x-1} * \dfrac{(1-\tau)C}{r} - Vb}{Vb}\right)\right)^2 \sigma^2 \\ D(V) = \dfrac{C}{r} * \left(1 - \left(\dfrac{V}{Vb}\right)^{-x}\right) + \left(\dfrac{V}{Vb}\right)^{-x} * \left((1-\alpha) * Vb\right) \\ E(V) = V - \dfrac{(1-\tau)C}{r} + \left(\dfrac{(1-\tau)C}{r} - Vb\right) * \left(\dfrac{V}{Vb}\right)^{-x} \end{cases}, (2.39)$$

With:

$$Vb = \frac{(1-\tau)C}{r + 0.5 * \sigma^2}, (2.40)$$

$$x = \frac{2r}{\sigma^2}, (2.41)$$

So here, from three equations with three unknowns, the parameters V, σ and α can be identified. However, unlike the previous system, this system of equations might not have strong-form numerical solution. Moreover, it can be stated, that there are quite few situations, where the

numerical solution can be found, so, the situation when the parameters of the model can be identified with solving this system, it, can serve additional proof that for the analyzed company most of the assumptions that were previously made are true.

On the other hand, since the model is only a reflection of a real life, some approximations are possible. So, if there is no numerical solution for this model that can be found, following procedure of identification model parameters will take place. In order to solve the system, firstly it is needed to identify estimation errors of the model:

$$Residual(\sigma_E)=\left(\frac{\sigma_{Eesimated}}{\sigma_E}-1\right)^2, (2.42)$$

$$Residual(E)=\left(\frac{E_{esimated}}{E}-1\right)^2, (2.43)$$

$$Residual(D)=\left(\frac{D_{esimated}}{D}-1\right)^2, (2.44)$$

After this, we can use the least residuals sum squares, in order to gain the best possible solution in terms of un biasness So, with minimizing the sum:

$$\sum_{¿} ¿$$

$$\sum_{¿} ¿$$

$$Residual ¿$$

And if there is a solution with all the residuals less than 2.5 $\%^2$ and their sum is also less than 2.5%, it will be assumed the found solution is close enough to be considered as the unbiased estimation. So, technically, identifying the parameters estimation is the task of function minimization, which can be done in Excel solver. The number of $2.5\%^2$ was taken as an 5% analog. Since 5% is common trust interval in the statistics, here it was also taken as a border condition. So, all in all, if a numerical solution for this problem is found, we assume that the parameters estimation are true, and, what is more, the assumptions necessary for the model holds in the particular case, since there is quite small amount of conditions, that lead to possibility of numerical solution.

2.2.2. Optimal leverage identification

After all the unknown parameters of the model were identified, the optimal leverage structure of the companies can be defined. The thing here is, that with rising the debt value, the required coupon value will also rise. What is more, with the rising coupon values, the level of assets, when the bankruptcy is triggered is also rising, so from the certain point of leverage, the values of leverage the value of debt is falling, as well as value of the firm. So basically, the value of the optimal leverage structure is identifying by the following procedure. Firstly, the

connection between leverage fraction and required coupon should be found. To found this function, it is needed to state that leverage fraction (L) can be defined as:

$$L=\frac{D(V)}{u(V)},(2.46)$$

Since, it was already mentioned that value of unlevered firm (V) does not depend on value of leverage, as well as the other values, like r, α,σ^2,τ, it could be said that the parameters that are changing with the value of leverage is coupon, and so debt value, and the equity value. So the fraction can be presented as:

$$L=\frac{D(C)}{D(C)+E(C)},(2.47)$$

So:

$$D(C)*\left(\frac{1-L}{L}\right)=E(C),(2.48)$$

or

$$\left(\frac{C\left(1-\left(\frac{V}{Vb}\right)^{-x}\right)}{r}+\left(\frac{V}{Vb}\right)^{-x}*((1-\alpha)*Vb)\right)\left(\frac{1-L}{L}\right)=V-\frac{(1-\tau)C}{r}+\left(\frac{(1-\tau)C}{r}-Vb\right)*\left(\frac{V}{Vb}\right)^{-x},(2.49)$$

After simplifying and adjustment the model can be presented as:

$$\left(\frac{\frac{1-L}{L}}{r}+K\right)=V-C^{X+1}\left(\frac{A*1-L}{L}-B\right),(2.50)$$

$$K=\frac{1-\tau}{r},(2.51)$$

$$A=\left((1-\alpha)Vbc-\frac{1}{r}\right)\left(\frac{Vbc}{V}\right)^{x},(2.52)$$

$$B=(K-Vbc)\left(\frac{Vbc}{V}\right)^{x},(2.53)$$

$$Vbc=\frac{1-\tau}{r+0.5\sigma^2},(2.54)$$

$$x=\frac{2r}{\sigma^2},(2.55)$$

Where everything except coupon (C) and value of leverage (L) are constants, so this model is a function of coupon from leverage level – C(L), but due to complication of the model it cannot be presented in the classical form. However, this model can be solved simultaneously.

So, in order to find optimal leverage fraction, we define the needed coupon for every leverage fraction, which represents the growing cost of debt for the company, since with higher leverage means higher risk of default, so the debt holders will require higher return on debt in

order to compensate the higher risks. The leverage value will be changed with the step of 0.1%, from $L_{0..100}$, and for every L_n the corresponding coupon $C_{0..100}$ will be defined through the way that previous equation will be true. After completing this procedure, there will be defined 1000 coupons, and each of them will correspond to the certain leverage fraction. Then, for each coupon will be calculated the value of the debt and the value of equity. Now, since

$$u(C)=D(C)+E(C),(2.56)$$

The maximum possible variable of u(C) will be found. The coupon that corresponds to this value, will be the "optimal" coupon for the firm, so after this the leverage value that corresponds to this coupon should be defined, so this leverage value can also be defined as optimal. Here, the optimal level of leverage, is a level of leverage, at which the company's value is maximized. Unlike other models of identifying optimal leverage structure, this model does not have a goal to minimize possible cost of capital, or risk of the company, since the value of risk as well as cost of the debt is already included in the calculations of overall company's value.

So, as it can be seen, the objective of this paper of developing a model of identification of optimal capital structure was completed. However, in order to gain better understanding of the model the analysis should be done, of how changes in different variables effects the choice of optimal leverage structure. It could give a clue, why the model suggests certain level of leverage for the one companies, and other one for other companies. Moreover, most of the changes should be explained from the logical point of view, in order to prove model's consistency. The most important part of this analysis is the analysis of changes in the optimal leverage, connected with different variables, since it can explain the whole logic of identifying optimal leverage structure for different companies.

Theoretically, the model performs in a following way:

Variable	*Shape*	σ^2	r	α	τ	*Call* (C_{naive}, 1)
Coupon (C)	Linear in V	< 0	> 0	< 0	> 0	< 0
Value of Debt (D)		< 0	> 0	< 0	> 0	< 0
Value of Equity (E)		> 0	< 0	>0	< 0	> 0
Asset value, that triggers bunkruptcy (Vb)		<0	> 0	< 0	> 0	< 0
Leverage (L)	Invariant to V	**<0**	**> 0**	**< 0**	**> 0**	**< 0**
Yield of debt (R-r)		> 0	< 0	< 0	>0	< 0

Table 2.2.1 Comparative statistics of financial variables at the optimal Leverage ratio

In the following table it is shown, how different variables change the nature of optimal leverage fraction. To begin with, it should be explained, that the yield of debt here, is the difference between the required return on debt for the optimal leverage structure and the risk

free rate. Of course, the changes described in the model are calculated with the assumptions, that all other factors, except the analyzed ones are staying the same.

The first changes are connected with the increasing of the firm's business volatility- the volatility of unlevered firm. If the volatility of firm increasing it means higher risks of the firm, so the optimal leverage structure will be decreasing, and the yield of the debt will grow higher, due to higher risks. As the yields are going up, the optimal coupon is going down, since, as it was explained earlier, the coupon payments are representation of required return on debt. The explanation of the falling optimal bankruptcy triggering asset value (Vb), is also quite understandable, due to the fact that higher possible changes in value of unlevered firm leads to the higher risks of it reaching the Vb, so in order to compensate it the optimal Vb value should decrease.

Increasing in the risk free rate, leads to the higher optimal leverage structure. This situation, that is surprising from one point of view, can be explained with the fact that the basic assumption of this changes is that every other parameter, except risk free rate remains the same. From the logical point of view, it means, that even though, the overall market becomes more risky, the company was able to keep the same amount of risks and returns, which means that from the point of view of overall market the company become less risky. So, now it is easy explained why in this case the model is suggesting to increase the level of leverage. The other thins here, is that the yields of the debt is decreasing, so the debt for the company is becoming cheaper, than it was before, so it also explains why in case of the rising risk-free rate the level of company's leverage should increase. On the other hand, in the real cases, rising of the risk free rate leads to the rising the risk of the company itself, since changed economic situation, so the previous results should be described with the caution.

The rising rate of recovery is causing quite obvious consequences. Since, the higher level of bankruptcy costs the debt holders are experiencing more risks, connected with the default, because in the case of default they will return less of their money. This situation leads to the higher required return on debt, to cover these additional possible losses, so the optimal leverage structure should be reduced, due to the higher cost of debt. So, for the firm with the higher losses of assets in case of bankruptcy the optimal level of debt will be lower, comparing with the firm, who will have lower α under the same circumstances.

Higher taxes, quite obviously means higher optimal leverage fraction, because of higher tax benefits from debt. So, if the taxes are rising and nothing else is changing, it is recommended to the firm to take more debt, to reduce the taxes paid.

The increasing in the price of "obligatory hedging" is basically the interpretation of the rising currency risks, associated with the debt. It can happen due to the decrease of the currency

profits, or the increase in the RUB/USD volatility, or both. In the case of the huge Russian companies, this value significantly increased, due to both mentioned reasons, since the oil prices went down, as well as rubble currency. This increase the value of the theoretical call option, that was implemented in the model as a value of currency risk of debt. If this call price is going up, the cost of debt is rising, since this price is included in the coupon payments, so the optimal level of leverage should be decreased, because the risk of bankruptcy is rising.

All in all, analysis of the model performance shown that every change in the model can be explained from the logical point of view. Some of the analyzed patterns will be used as an explanation for the observed results in the next chapter. So, now, when the model is build and described, it should be tested on the real cases, in order to analyze its performance, make conclusion about its accuracy, and fulfilling the last objective of the thesis.

To summarize the develop model, there algorithm of its' implementation to the real company was designed. In order to implement the developed model, for every tested company the following algorithm should be used:

1. Determine the proxies of the model for the tested time-moment
2. Estimate parameters without currency risk
3. Fond the cost of hedging for the tested time-moment
4. Adjust coupon, in order to take into account the currency risk associated with debt
5. Estimate parameters with the adjusted coupon
6. If there is no closed form solution, use the minimum sum of squared residuals method
7. If there is a solution gained with the minimum sum of squared residuals method define coupon values at all values of leverage
8. Find a coupon that maximizes total value of the firm
9. Determine leverage value for this coupon
10. Analyze observed results, determine all the meaningful values for the found solution

So, in the last chapter, this algorithm will be implemented to different companies in the different time moments.

3. MODEL IMPLEMENTATION

3.1. Russian crisis 2014-2015 description

One objective of the model is to show differences between Russian companies' cases in two different conditions. The first part is relatively stable economic situation that can be described by reliable growth in GDP. To identify utility of the designed model, comparison of cases in conditions of stable economy with cases of crisis will be implemented. Degree of differences between these two options is presented in Table 3.1.1.

Period	A. 2008/2009 crisis	B. 2009/2011 recovery	C. Pre-crisis Q3 2014 – Q1 2015
GDP growth	-7.8%	4.9%	0.6%
Oil price	$116 to $42	$42 to $112	$103 to $53
Foreign borrowing (net change in foreign debt)	Falls $85 bn	Rises $63 bn	Falls $100 bn
Reserves of central bank	Falls $222 bn	Rises $123 bn	Falls $161 bn

Table 3.1.1 Features of Russian economy in different time periods, [Gregory 2015]

However, crisis shows that international reserves of Russian Federation that consist of various types of foreign assets has significant drop [Mchugh, 2015]. This evidence represents that coming out of recession requires essential funds. Nevertheless, it is hard to admit positive changes in economic situation since 2014.

The most considerable impact on economy was done by implementation of US and European sanctions that restricted access for several Russian companies to international financial resources. Reason for sanctions was "annexation" of the Crimea and supposed support of separatists in eastern Ukraine. Consequently, shortage of sufficient funds and investors outflow because of high uncertainty in the market contributed to fall of the ruble exchange rate.

This is another significant problem that worsened conditions for the Russian companies. Because of devaluation of ruble more than 50%, overall costs of operations increased, especially, in cases where companies use imported material for production or service [Hobson, 2015]. Number of organizations had bank loans and issued debts denominated in foreign currencies. Those who do not have earning in foreign currencies faced problem of debts' return.

Additional effect created drop in oil prices as Russia has one of the biggest volume of oil export in the world. The prices of oils dropped dramatically during seven months – from June to December. The price was changing from 100 US dollars per barrel to the level below 50 US dollars [Bowler, 2015]. The main reason of these circumstances is decline in demand all over the world while producers increased volumes. Almost the half of federal budget of Russian

Federation consists of oil and gas sales. For the last two decades, national specialization of Russia was in extraction and export of natural resources. Without strong diversification in different industries except natural resources production, it was almost impossible to minimize effect of crisis on economy. Even during preparation of budget for 2014, government used oil price equaled to $100 per barrel. Therefore, execution of suggested budget was complicated because of prices' fall twice. Implemented solution that was aimed to fulfil shortage in budget was to increase production and export of oil.

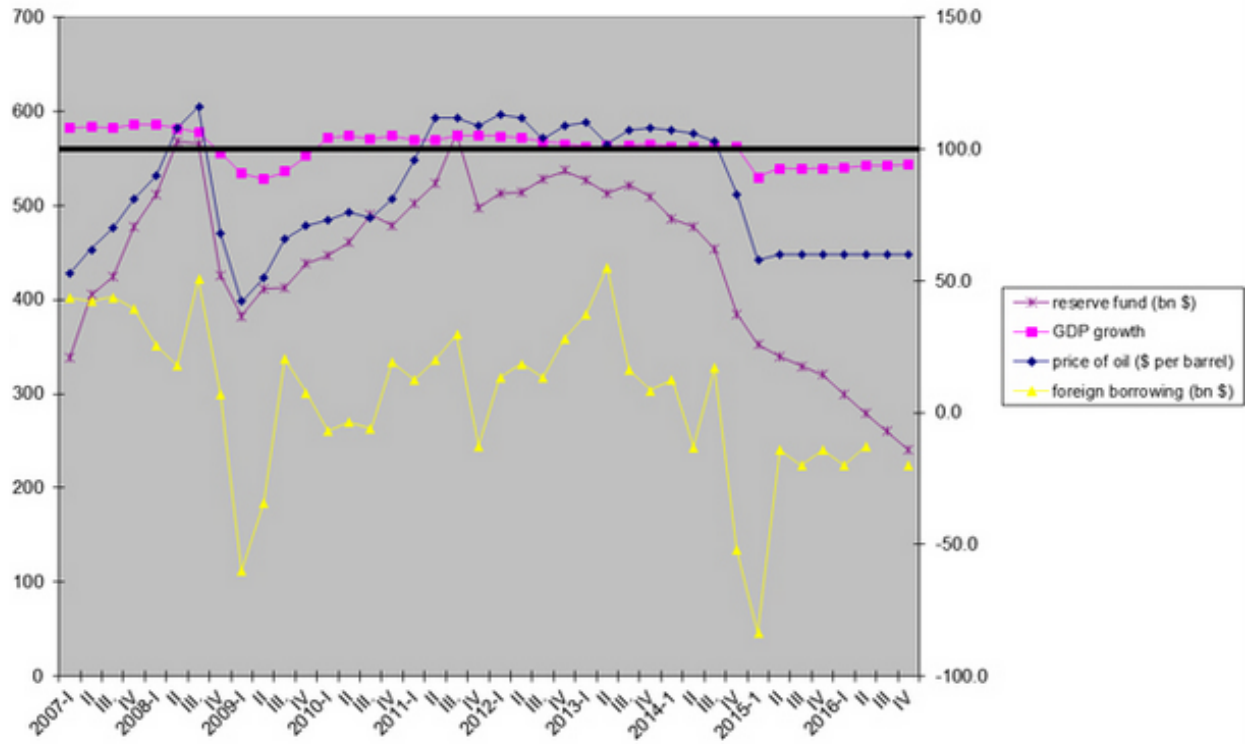


Figure 3.1.1 Russian GDP growth, oil prices, foreign borrowings and reserve fund over 2007-2016, [Gregory, 2015]

Figure 3.1.1 shows even during 2007-2012 high volatility in these indices existed. However, general tendency from crisis 2008-2009 represented gradual increase in oil prices and reserved funds that indicates high revenue of the Russian Federation budget. Moreover, GDP was rather stable and companies were able to borrow money in foreign markets to cover internal shortage of funds. Period from the second quarter of 2013 is crucial moment for reserve funds as access to foreign loans were denied for several government-related companies. For the following years list of companies expended and the first quarter of 2016 showed the lowest level of foreign borrowings. However, from this period government intended on convergence with China that allowed to cover shortage of funds partly. In addition, price of oil dropped significantly that collapsed Russian GDP. The only source of significant resources was reserve fund that was used by the government to support key industries.

3.2. Investigated companies' profiles

3.2.1. Lukoil

Lukoil is Russian petroleum company and the second largest after Gazprom on revenue volumes in Russia. The interesting fact that previously Lukoil was the largest organization of Russian oil industry on production volumes till 2007, when Rosneft replaced it.

The company were established in 1991 and combined several Siberian organizations. In 1992 it was transformed in open joint stock company by president order. Since 1996 Lukoil began to finance operations with a help of ADR and was able to begin construction of own tanker fleet for maritime transportation. In addition, company did range of acquisitions of large oil companies in 1999.

Beginning of 2000s were marked by the first international acquisition of the company with purchase of Getty Petroleum Marketing Inc. The crucial moment of Lukoil's history was sale of last government owned shares of the company to ConocoPhillips. The valuable part of strategy was not only acquisitions strategy, but also creation of joint enterprises with such companies as Gazprom in 2007 and ERG in 2008. Cooperation with international companies was utilization of common advantages for Lukoil. For example, organization participated with Statoil in tender for exploration of Iranian oil fields and won it.

General concentration of Lukoil is petroleum industry and related activities: exploration, extraction, refinement, transportation and sales of petroleum and gas products. However, Lukoil has diversified in related businesses like electrical power by acquiring numerous number power plants all around Russian. Moreover, except B2B segment Lukoil developed network of gas station and sales petrol to end customers.

3.2.2 Rosneft

Rosneft is current leader of Russian oil market and estimated as the largest traded on the stock exchange oil company in the world. However, it is government ruled company as government is the owner of big stake in Rosneft's equity. Main concentration of the company is exploration, extraction, refinement and sales of petroleum products. The geography of companies' operation covers major regions of Russia. Core stream of sales is export.

Rosneft was founded in 1993 on the base of another enterprise – Rosneftegaz, which was the core petroleum organization in Soviet Union. The interesting fact is that for the purpose of avoiding monopolization ten subsidiaries of the company were excluded and established as

independent organizations. However, over time several of them were consolidated again in a form of acquisition. The owner of Rosneft, which is strategic enterprise even nowadays, was government. Nevertheless, in 1995 solution was accepted to change legal form of organization to open joint stock company.

Last years of 1990s can be characterized as reorganization of the company with previously ineffective management and low outcome of production assets. The next stage were connected with intention of Rosneft's privatization. Assets of large oil company attracted different big representatives of the industry. One possible and the most probable buyer was Sibneft company; nevertheless, transaction didn't take place because alliance of Russian oil companies made own proposal. This competition led to cancellation of privatization.

The beginning of 2000s were connected with management activities directed to consolidation of the organization's assets, decrease of leverage appeared during Russian crisis of 1998 and purchasing of licenses for oil deposits. During this decade organization acquired several big Russian oil companies and expanded geography of extraction. Moreover, Rosneft began to acquire foreign entities and to strengthen relationships with large international organizations. In 2001 there was proposal of merger with Gazprom, which, however, were not accepted by the representatives of the companies.

One of the most significant event that is connected to acquisition policy of Rosneft was in 2013. This year Rosneft acquired 50% of joint venture TNK-BP, which made Rosneft the largest public oil company in the world [Rapoza, 2013]. According to annual financial reports for 2015, the last year acquisitions include the following events:

- Rosneft acquired AET Raffineriebeteiligungsgesellschaft mbH by purchasing 66.67%.
- Acquisition of 100% ownerships were utilized for LLC Trican Well Service (TWS), Petrol Market Company, Novokuibyshevsk Petrochemical Company, Orenburg Drilling Company, Bishkek Oil Company.
- Rosneft acquired 8 enterprises of Venezuelan Weatherford International plc.

In 2016 Rosneft capitalization exceeded capitalization of Gazprom that gives opportunity to call Rosneft the most valuable company of Russia [Bierman, 2016].

Figure 3.2.1. Rosneft (blue) and Gazprom (white) capitalization, (Bloomberg)

3.2.3. Novatek

Novatek OFSC is Russian public company that has shares that are traded in Moscow and London stock exchanges. The company is the representative of gas industry and the second large producer of natural gas in Russian according to production volumes. The interesting fact is that Novatek is 6th company in the world in volume production, whereas its costs on exploration and development of deposits are one of the lowest among gas producers. According to annual report for 2015, the organization has share of 20% in gas supply of Russian market.

Main concentration of the company is exploration, extraction, conversion, transportation and sales of natural gas. The core facilities of Novatek are situated in Yamalo-Nenets Autonomous District. All produced gas is sold in Russian market. Therefore, company tends to enter international markets by creation of new sales channels.

Company Novafininvest was found in 1994 and had concentration in the field of oil and gas. Further, its name was changed to Novatek in 2003. As it was established in Yamalo-Nenets Autonomous District, company acquired direct access to activities in oil and gas industry by purchasing of licenses for two local deposits. Implementation of the projects on the development and production of gas required considerable investments in infrastructure and exploration itself.

Moreover, exploration and following development of field required a lot of time. Consequently, initial steps began in 1996 which led to the first sales only in 2002. From this point company started to sell gas-condensate in the market. The following years was connected with optimization of business which caused to restructuring. Organization began to differentiate core businesses with non-related and commenced to dispose non-core assets to focus on oil and gas industry.

One of the largest and the most important events of new Novatek was launching of the Purovsky Plant that became core organization's production facility. In the same 2005 company decided to become public and conducted IPO on two stock exchanges – London stock exchange and further Moscow stock exchange.

According to annual financial reports, the latest (considering two previous years) significant strategic solutions concerning investments, acquisitions and disposals include the following events:

- Joint company of Novatek and Gazprom acquired 60% of Artic Russia B.V. in 2013. Simultaneously, Novatek made an agreement with Rosneft for exchange of 51% of Sibneftegas to acquire 40% of Artic Russia B.V.
- Novatek sold 20% of equity shares of Yamal LNG to China National Petroleum Corp. in 2013
- Novatek acquired additional share in LLC Nortgas increasing its ownership from 49% to 50% in 2013.
- Novatek gained share of 100% in LLC Novatek-Kostroma by increasing shares number on 15% in 2014.
- Novatek acquired 100% of JSC Office's shares in 2014.
- Novatek acquired LLC NovaEnergo by purchase of 100% of the company in 2014.
- Novatek reduce groups' ownership share in Artic Russia B.V. by disposal of 20% in 2014.
- Novatek sold 9.9% in Yamal LNG to Chinese fund in 2015; whereas remains share of 50.1%,which gives ownership of the project in cooperation with Total SA and China National Petroleum Corp.

One of the latest significant events that highly affected operations of Novatek was inclusion of the company into sanction list of the Office of Foreign Asset Control [U.S. Department of the Treasury, 2014]. This organization is connected directly with the US Government. Inclusion in this list means that company has no access to long-term debts in western financial organizations. The following situation was caused by political isolation of the Russian Federation concerning to Crimea. Novatek was included as representative of the Russian Energy Sector, although company has no operations in Ukraine. However, this affected current

operations of Novatek significantly. New projects of liquid natural gas deposits development in Yamal suffer seriously from lack of funds [The Moscow Times, 2014]. As these projects were developed on cooperation with foreign partners – Total SA and China National Petroleum Corp. – companies had to seek fund in China financial market [Marson and Williams, 2015].

The joint project with Russian government and several companies was aimed to develop port Sabetta to the ability to export by sea. However, in conditions of budgets' shortage Novatek had to sell share of equity in Yamal LNG to Chinese investment funds. According to consolidated interim condensed financial statements for the first three months of 2016, company disposed of 9.9% of share to support activities of the project. In general, Novatek and its subsidiaries use the following ways of finance:

- Internal borrowings.

Companies ruled by the shareholders' agreement are able to provide different subsidiaries with internal credit lines both in rubles and in US dollars.

- Eurobonds.

The group of companies issued Eurobonds with different termination dates.

- Bank loans.

Novatek has available in Russian bank credit lines for long-term and short-term periods. Each opportunity has negotiable interest rate and temporary restricted contract for the implementation of the credit line.

3.3. Model implementation

3.3.1. Case of Lukoil company

Firstly, as it was described in the proposed algorithm it is needed to estimate the proxy values for every year. The information about the company was taken from the company reports for the respective years. The information about capitalization and volatility of the company's equity was taken from the open sources, since its market information, that is publicly available. The risk-free rates for the certain years, as well as the information about currency changes were taken from the central bank of Russia.

Lukoil Company	Year			
Proxies	*2010*	*2013*	*2014*	*2015*
Value of Debt (mln rubles)	338 520	351 324	756 118	859 713
Risk-free rate	8.40%	7.40%	12.00%	11.40%
Value of equity (mln rubles)	719241	1208830	815902	1058749
Year Volatility of the company	23.51%	22.28%	38.57%	41.65%

Table 3.3.1. The proxy values for Lukoil company

It can be seen, that the difference between pre-crisis and crisis years is quite significant. Firstly, the value of debt had been raised in 2014 more than twice, comparing to 2013. This had happened due to the fact, that huge amount of Lukoils debt were issued in the form of Eurobonds, so when the dollar strengthen almost double time against rubble in 2014, the value of debt corresponded to it. The risk free rate was also increasing, since the Russian economy suffered a lot from the crisis of 2014, and long term rates on the Russian federal bonds, had increased almost for 50%. Due to all this factors, the yearly volatility of the company also rise a lot, being almost twice as much in 2015 as it were in 2013. It also should be mentioned that, the difference between the years in the same category (pre-crisis and crisis), does not differs much, however, the difference between this two categories is huge. So, it can be said that, this environment is a perfect field for model testing, since here are both situations: similar looking years, and highly different, so any gained results can be analyzed in deferent circumstances, but at the same time, can be compared in the similar situations. Now, according to the algorithm of model implementation the parameters of the model should be estimated by solving the problem of minimization the sum of residuals. By doing this, the following results were gained:

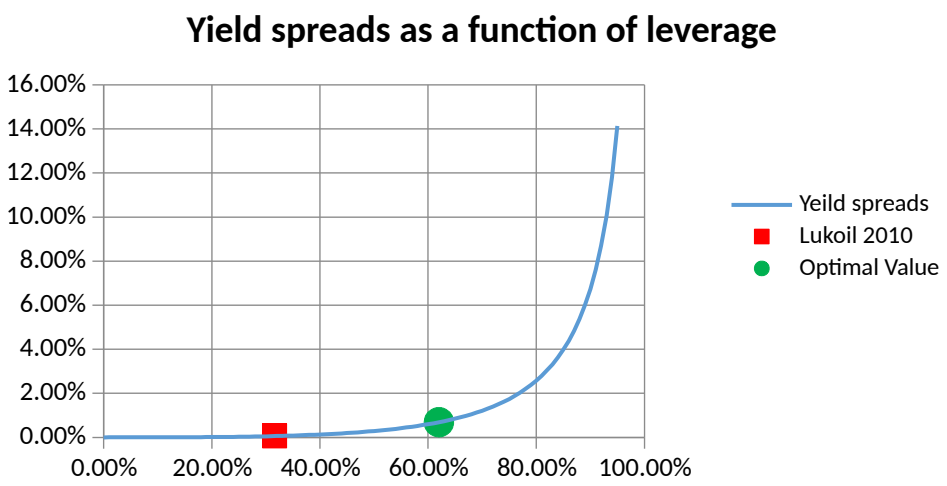
Lukoil Company	Year			
Estimated Parameters	*2010*	*2013*	*2014*	*2015*
Value of business (mln rubles)	986169.07	1485544	1530210.75	1862044
Volatility of business	24%	25%	43%	44%
Rate of recovery	91%	91%	93%	90%
Effective Tax Rate	25%	25%	25%	25%
Risk free rate	8.40%	7.40%	12.00%	11.40%
Coupon (mln rubles)	28953.11	27381.3	109708.8942	117669.7
Required return on debt	8.55%	7.79%	14.51%	13.69%
Fraction of uncovered currency debt	85.3%	92%	83.2%	83.1%
Price of hedging as a fraction of coupon	5.3%	4.53%	23.5%	15.3%

Table 3.3.2. Estimated parameters for the Lukoil company

So, here it can be seen that the same patterns as for the proxy values appears in the estimated parameters. First thing, that should be mentioned, is that through the years the value of unlevered firm is constantly rising, which can be considered as a proof that Lukoil is quite successful company. The growth continues even in the crisis years, so it can be assumed that the company is managed in a proper way. However, the volatility of unlevered business grew significantly in the crisis years. Again, here can be seen a pattern, that the situation with volatility is quite similar in both groups of years, which partly proves the assumption about constant volatility of unlevered firm. So, already can be defined, that some of the assumptions of the model holds, but with huge changes in the circumstances, the values are also significantly

changes. The recovery rate, however, stays the same for all years, despite the crisis, so the conclusion might appear, that recovery rate does not depend on the external market situation, and this thought is quite logical, since recovery rate is mostly characteristic of the company and its possibility to cover possible bankruptcy costs. The effective tax rate also stays the same trough all the years, which also is understandable, since tax rate depends on the tax codes of the countries, where the company is paying taxes, but not on the market conditions. The coupon that Lukoil is paying for the debt is rising with the crisis years and this has several possible explanations. Firstly, the falling market and prices of oil made Lukoil more risky company for investment, as well as its risen volatility. The other reason is again the ruble falling, since most of the Lukoils obligations were in currency, the amount of rubles paid for the interest had risen. The same situation is with return on debt, since it is a representation of coupon. What is more, it can be seen that the price of "hedging" for Lukoil, as a value of a currency risk rose significantly. This happened because higher volatility, and lower currency profits, so in 2014 and 2015 the cost of debt had become significantly higher for the Lukoil. Now, when all parameters are estimated, the optimal leverage structure of the firm can be defined for different years.

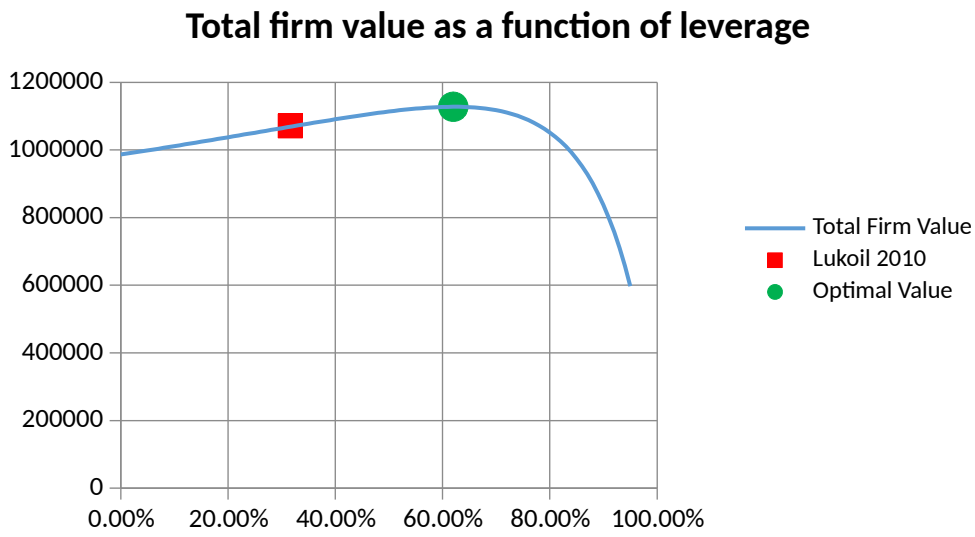
Firstly, yield spreads associated with the choice of leverage should be analyzed:



Graph 3.3.1. Yield spreads on debt as a function of leverage

As it can be seen from the graph, the debt was really "cheap" for Lukoil in 2010, that's why, the model suggests for Lukoil in 2010 to take more debt, in order to benefit from the tax shields.

From the point of view from the overall firms value, the situation is the following:



Graph 3.3.2. Total firm value as a function of leverage

So, it seems that because of low required rate on return on debt for Lukoil in 2010, the model assumes that the leverage number should be increased. This can be explained with the fact, the the model mostly includes only two risks: the risk of default, and currency risk. However, the company suffers from other possible risks, that are not fully included in the model. But still, as it will be seen further, the model is quite accurate in terms of identifying leverage structure under certain circumstances. From the point of view of numbers, the situation for Lukoil in 2010 is described in the following table:

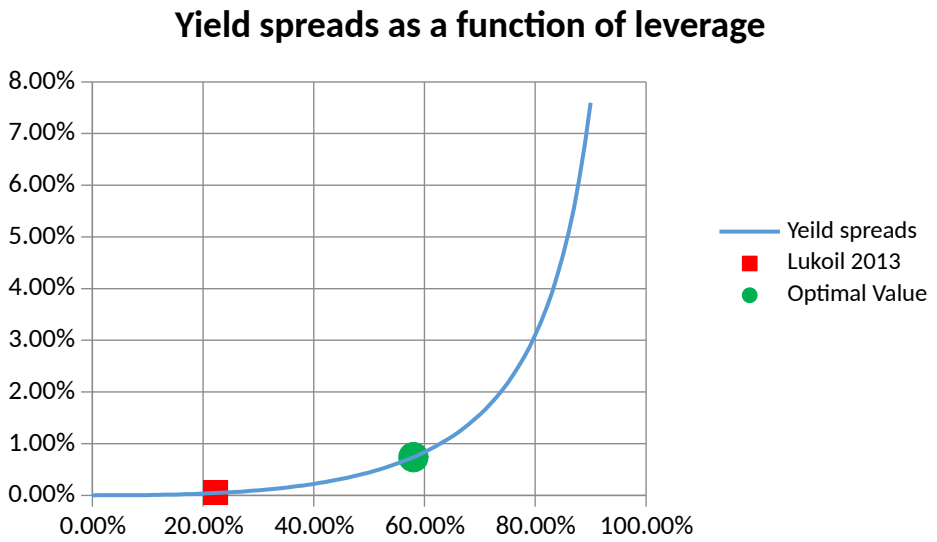
Values	*Lukoil 2010*	*Optimal Value*	*Change*
Leverage	32%	62%	96%
Debt	338520	699051.75	107%
Coupon	28953.11	63523.97	119%
Requared return on debt	8.55%	9.09%	6%
Yield spreads	0.15%	0.69%	350%
Total Firm Value	1071050.1	1127502.83	5%
V-Bankruptcy	194967.04	427763.40	119%
V-Bancruptcy/ Total Firm Value	18%	38%	108%

Table 3.3.3 The optimal leverage rate for Lukoil 2010

Here it is shown that increasing level of leverage would increase the total company value. However, Huge amount of risk appears, but the value that can be added with this choice of leverage is only 5% from the total value of the Lukoil, so it might be rational managerial decision not to increase the leverage, because outcomes just do not cover the risks, since, as it was mentioned already, the model is limited by its assumptions. Moreover, in this table there is a

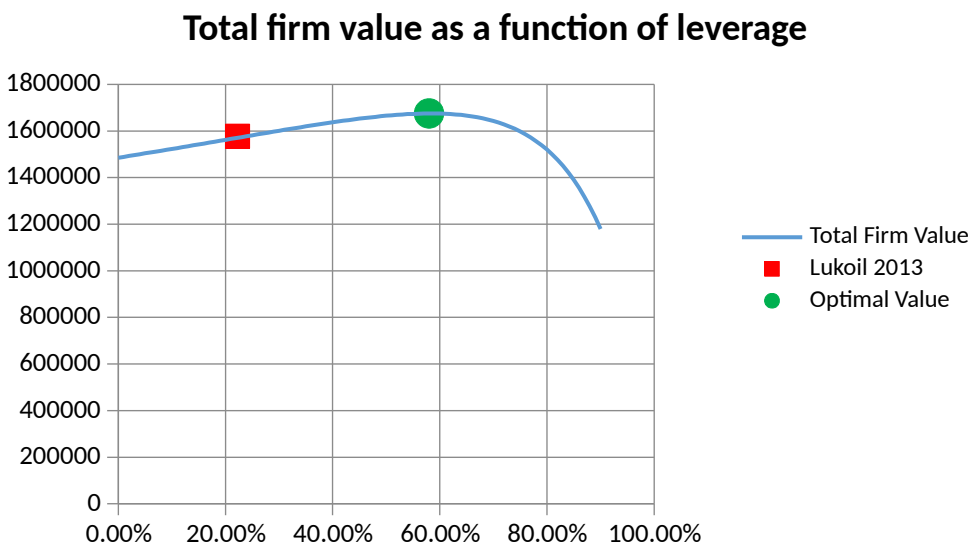
new parameter of the firm introduced: fraction of Vb to the firms value. This can be an interpretation of a risk of the firm, because the higher percentage of the firms bankruptcy triggering value is from the value of firm, the more probability the firm has to become bankrupt. Of course, due to the complicated nature of the option-like nature of firms equity, this fraction cannot be easily interpreted, it still can serve as a certain indicator of the firms bankruptcy risk.

The situation for the 2013 year is really common to those of 2010. The yield spreads are:



Graph 3.3.3. Yield spreads on debt as a function of leverage

So, the same as in 2013, the "cheap" debt is leading to the recommendation, made by the model to take more debt in order to have more tax benefits. The situation with the value of the firm, is also really alike to 2010:



Graph 3.3.4. Total firm value as a function of leverage

The reasons, for the same recommendations as were made in 2010 is the same, since the market situation and economic conditions are nearly the same. Even, the overall company value

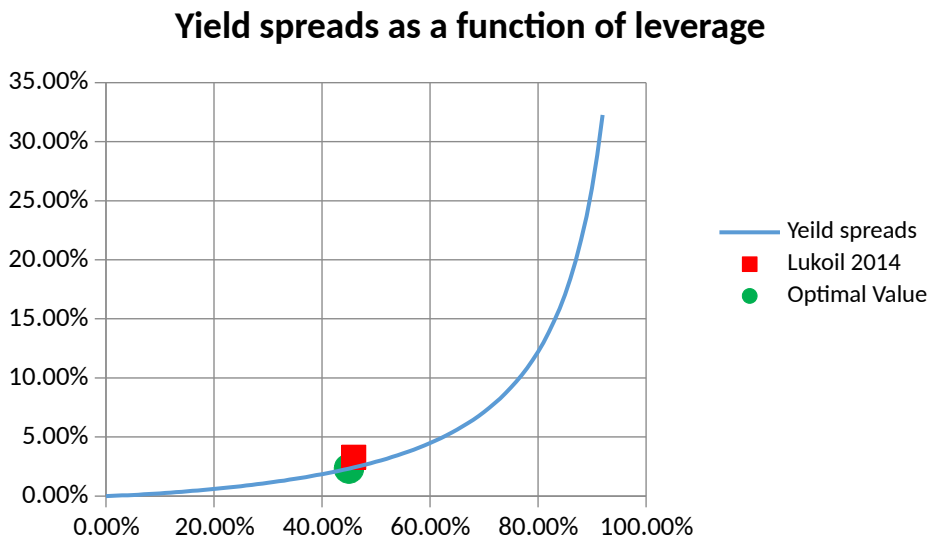
grew almost for a 40%, the amount of recommended leverage didn't change much. This situation partly proves the assumption, that leverage value does not depend on value of business. Moreover, it proves that, if the market situation doesn't change significantly, the model gives the same results, which partly proves the assumption of time-independence of the optimal leverage structure. In the following table numerical solution for 2013 is presented:

Values	*Lukoil 2013*	*Optimal Value*	*Change*
Leverage	22%	58%	161%
Debt	351324	971827	177%
Coupon	27381.309	79377	190%
Requared return on debt	7.79%	8.17%	5%
Yield spreads	0.36%	0.74%	103%
Total Firm Value	1578401.5	1675564	6%
V-Bankruptcy	280008.54	572077	104%
V-Bancruptcy/ Total Firm Value	18%	34%	92%

Table 3.3.4 The optimal leverage rate for Lukoil 2013

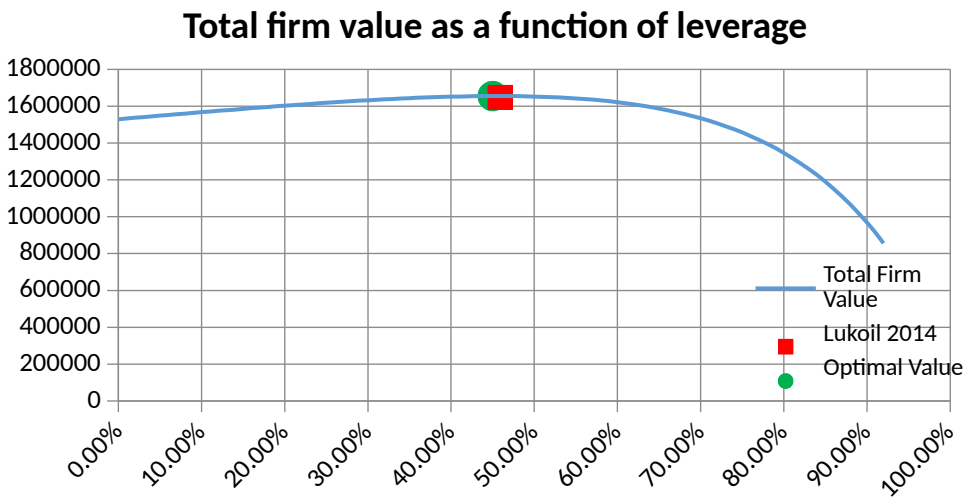
The same as for 2010, significant increase in leverage, and so in the risk of the firm will lead to the slight increase of the total firms value. So, again the difference in the model solution and a real situation most likely can be explained from the point of view of managerial decisions. It is recommended to the Lukoil, to increase its leverage ratio, because the ability to take cheap debt (because the really low default risk of the company) will pay –off greatly through tax benefits. The most important thing here, is that the results partly proves that the assumptions of time-independence of the optimal leverage structure for the same company, under alike conditions. The next step is to analyze crisis years, and the differences between results for different situations for the same company.

As it was described previously, the 2014 is really different from 2013 in terms of economical and market situation for the Russian companies. 2014 is a first crisis year for Russia, as well as for Lukoil. The currency risk risen, and the oil prices fell down, so there is a significant change in the Lukoils performance and model results. The yield function for Lukoil in 2014 is following:



Graph 3.3.5. Yield spreads on debt as a function of leverage

In this graph, it is clear that the debt is not so "cheap" as it used to be in 2010 and 2013. This had happened due to the fact that the overall risk of the company had risen, as well as the currency risks associated with debt. Also, the yield curve become more sharply, since the function parameters had changed, due to changed economic environment. The total value of the firm function also differs a lot from the situation in 2010 and 2013:



Graph 3.3.6. Total firm value as a function of leverage

So, the shape of the firms value curve had changed, and now optimal leverage structure predicted by the model, and the actual Lukoil capital structure are the same. This happened due to the fact that the reprising of the Lukoil debt, which was nominated in rubbles, but had to be paid in currency. Moreover, the overall company risk had rise, so the optimal capital structure predicted by the model was reduced. So, all in all, that situation represented the case, when the theoretical solution of a problem was completely approved on practice, so it can be said that

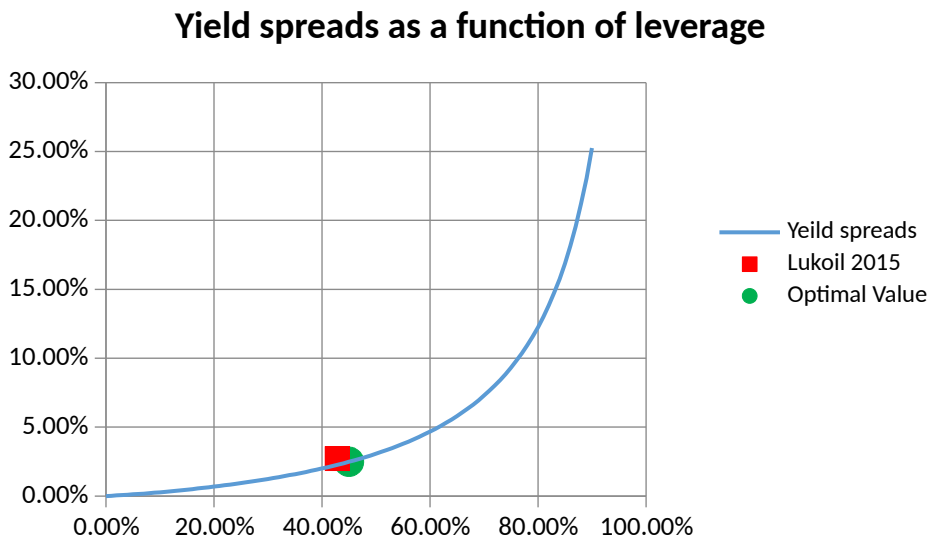
most of the assumptions that were made in the process of model development seems to be holding for Lukoil in 2014. The numerical representation is:

Values	*Lukoil 2014*	*Optimal Value*	*Change*
Leverage	46%	45%	-2%
Debt	756118	745006.4532	-1%
Coupon	109708.8942	106533.1371	-3%
Requared return on debt	14.51%	14.30%	-1%
Yield spreads	2.53%	2.32%	-8%
Total Firm Value	1648212.982	1655569.896	0%
V-Bankruptcy	385277.5179	374124.8413	-3%
V-Bancruptcy/ Total Firm Value	23%	23%	-3%

Table 3.3.5 The optimal leverage rate for Lukoil 2014

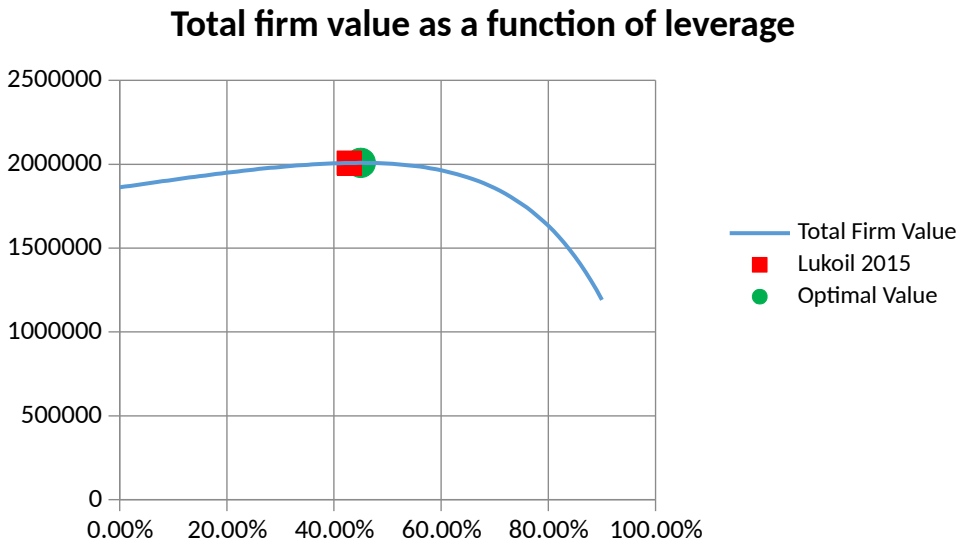
So, from this table it can be defined that the optimal leverage structure recommend by the model is really close to the capital structure that really was in Lukoil in 2015. However, this results are quite different from those in 2013. This can be explained by heavily changed economic conditions. The goal of analysis pre-crisis years and crises years was achieved, since gained solution corresponds to common sense, because when the crisis appears, the model recommends significantly lower leverage level than before the crisis, due to increase in risks. Finally, in order to complete study about the Lukoil Company, another crisis year should be analyzed, in order to prove the previously made assumption, that the optimal leverage structure is time – independent, under the same circumstances for crisis situations.

The situation in 2015 for Lukoil didn't change much, however, it should be stated that in 2015 risk of the company is less than in 2014. It can be explained firstly, with the lower risk free rate for this year, and secondly, with the lower price of obligatory hedging, since the volatility of the currency had become lower. So, the yields for the year 2015 are:



Graph 3.3.7. Yield spreads on debt as a function of leverage

It can be seen that yield spreads are nearly the same as for 2014 year, but the curve is slightly less sharp. This curve is still really different from the curves of pre-crisis years, which can explain the differences in the suggested capital structure. The value of the firm curve, is also looking the same as one for the year 2014, and quite differs from the pre-crisis years.



Graph 3.3.8. Total firm value as a function of leverage

So, as it can be seen on the graph, the same amount of leverage is recommended as it were in 2014. This can be explained by the nature of the proxies and estimated parameters, since they are the same for the crisis years. The numerical representation will be:

Values	*Lukoil 2015*	*Optimal Value*	*Change*
Leverage	43%	45%	5%
Debt	859713	903748.4	5%
Coupon	117669.7	125022.5	6%
Requared return on debt	13.69%	13.83%	1%
Yield spreads	2.34%	2.48%	6%
Total Firm Value	2007509	2008330	0%
V-Bankruptcy	421566.3	447908.7	6%
V-Bancruptcy/ Total Firm Value	21%	22%	6%

Table 3.3.6 The optimal leverage rate for Lukoil 2015

However, the optimal level of leverage is the same as a year before – 45%. And it should be stated, that Lukoil company suits this optimal level quite well.

To sum up, it could be said that the crisis years for Lukoil changed everything in terms of effective leverage. From tone point of view, the company became riskier, which is not really greal for its stakeholders. On the other hand, crisis years allowed it to achieve optimal level of leverage in terms of the proposed model. It is important that the model gives the same values, for the similar years. It can lead to the conclusion that the assumption of the time independency of the optimal leverage structure and overall model parameters can be considered as true, but only if the economic situation does not change significantly. As a managerial application, it can be stated, that the analysis of the Lukoil's capital structure defined that according to the proposed model, Lukoil's management had made right decisions in terms of capital structure, gaining all possible tax benefits, with the reasonable amount of default risk. However, it should be taken into account, that model itself has tendency to over valuate debt and its benefits, since the risks that are taken into account seems to be not the all risks that are associated with the firm. Still, the results shown are quite valuable for Lukoil, as well as for scientific community, since they show the theoretical behavior of the model, as well as its possibility to evaluate the risk value.

After gaining this results, it is needed to analyze, how the model performs on a basis of other similar company, that fulfill the model assumptions, in order to gain more variable testing results.

3.3.2. Case of Rosneft company

Firstly, as it was described in the proposed algorithm it is needed to estimate the proxy values for every year. The information about the company was taken from the same sources as for Lukoil company. The proxies are following:

Rosneft Company	Year			
Proxies	*2010*	*2013*	*2014*	*2015*
Value of Debt (mln rubles)	641 519	2 076 000	2 240 000	2 822 000
Risk-free rate	8.40%	7.40%	12.00%	11.4%
Value of equity (mln rubles)	1593213	304668.7	128374.8	250411.7
Year Volatility of the company	30.99%	20.83%	30.21%	28.01%

Table 3.3.7. The proxy values for Rosneft company

From this table, it can be defined that Rosneft in many cases is quite similar to Lukoil. However, the most interesting difference, and, basically a reason, why this company was also taken for an estimation is the fact that even in the pre-crisis years the company seemed to be more riskier than Lukoil. So, in order to gain better understanding about the model performance, there were taken two similar companies, but with quite different amount of risk. What is also interesting about the proxies of the rosneft, is that unlike the Lukoils debt, the debt of Rosneft had been risen not only in crisis years, but in the pre-crisis also. Of course, when the crisis appears, the value of Rosneft's debt risen even more, for the same reasons as for Lukoil – the fall of the oil prices, and the rubble currency. The risk free rate is the same for both companies, since it was proxied by the Russian federal long term bonds rates. So now, the same algorithm as was used for the Lukoil will be implemented, so the parameters will be estimated.

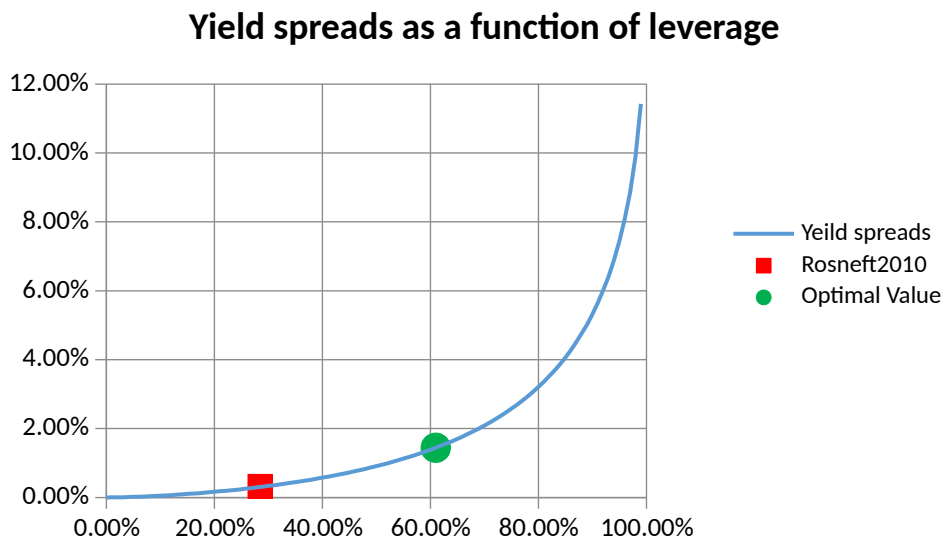
Rosneft Company	Year			
Estimated Parameters	*2010*	*2013*	*2014*	*2015*
Value of business	2131675	2754908	2610899	3051294
Volatility of business	33%	36%	81%	65%
Rate of recovery	30%	37%	29%	21%
Effective Tax Rate	20%	20%	28%	26%
Risk free rate	8.40%	7.40%	12.00%	11.4%
Coupon	54765.06	323249	1512577	920269.2
Requared return on debt	8.54%	15.57%	67.53%	33%
Fraction of uncovered currency debt	83.20%	79.90%	83.50%	94%
Price of hedging as a fraction of coupon	5.20%	4.30%	23.55%	18.70%

Table 3.3.8 Estimated parameters for the Rosneft company

From the table it can be defined that, even thought the proxy values were quite the same for the Rosneft and Lukoil companies, the parameters of their models differs a lot, despite effective tax rates, and price of hedging. The similarities can be explained by the fact that both companies operate in the same markets, so they pay alike taxes, and gain similar currency profits. However, the volatility of business of the Rosneft is higher than volatility of the Lukoil Company, which can be explained by the different business models that are implemented in the companies. Moreover, unlike the Lukoil company, who was continuously growing – in terms of unlevered firm value – even when the crisis appears, the Rosneft had decrease in value of

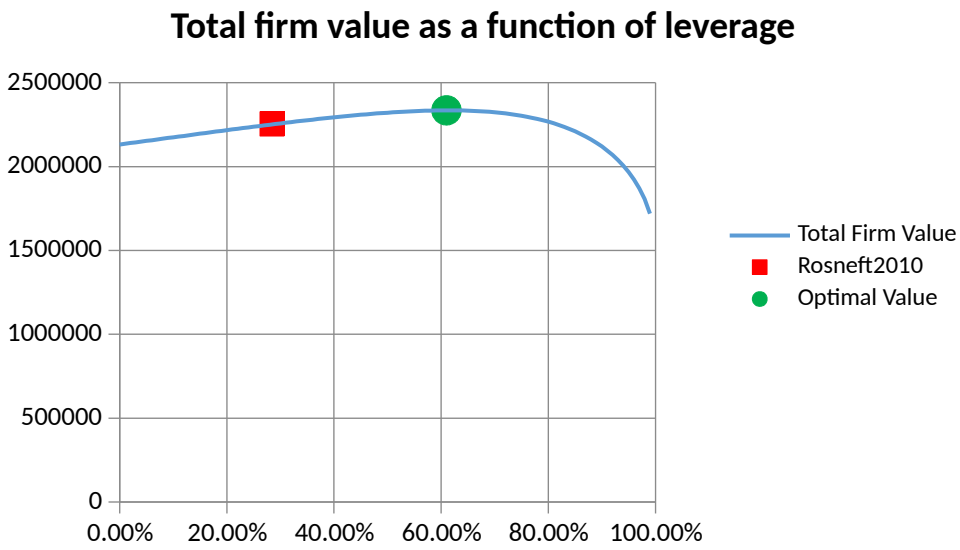
business in the first crisis year. This, most likely, was caused by the higher volatility of Rosneft business. Nevertheless, in the second crisis year, Rosneft business value was risen, and not only fully recovered from the losses, but gained some additional value compared to the pre-crisis years. As well as the situation with loss in value in 2013, the situation of huge addition to the value of business can also be explained by the higher overall volatility of the firm, since higher volatility means not only higher possible losses, but also the higher gains. This reflects the overall market rule: more risk is taken - higher possible profits can be gained. What is more, it is necessary to state that improving the unlevered value of capital is the common for every existing firm. If the firm does not improve its business value, it means that it doesn't creates any value, or, even working with losses of assets. This kind of situation can occur only for short period of time, since if this pattern continues to long, firm is becoming bankrupt, since its operations does not create any value. In terms of the proposed model, if firm has constantly reducing value of unlevered business, it reaches the value that triggers bankruptcy, and firm declares default. All in all, the difference between this two companies were analyzed, so now the differences im model implementation should be analyzed.

For the year 2010 the yields on debt for the Rosneft are following:



Graph 3.3.9 Yield spreads on debt as a function of leverage

This graph is really similar to the yield spreads for Lukoil company in 2010. This happened, since the values for both companies are really similar in this year. However, the debt is even "cheaper" for Rosneft, than it was for Lukoil at the same time period, since Rosneft have better rate of recovery. So, like it was described in the last paragraph of the previous chapter, this leads to the higher optimal leverage level defined by a model for the company. The situation with the firms value is also similar to the situation taken place for Lukoil at 2010.



Graph 3.3.10 Total firm value as a function of leverage

The situation here is really similar to the lukoil company for the same time period, with the difference mainly caused by other recovery rate. The numerical solution to this problem can be described as

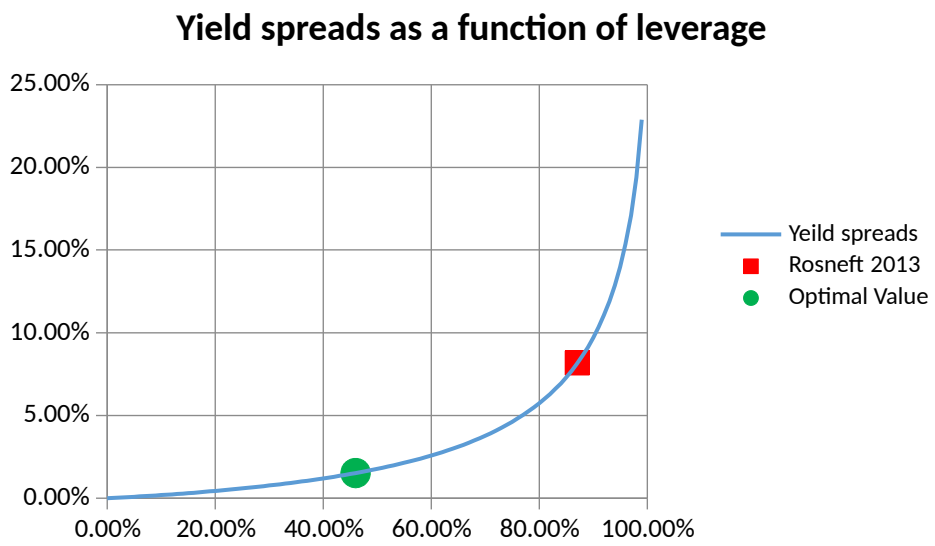
Values	*RosNeft 2010*	*Optimal Value*	*Change*
Leverage	28%	61%	114%
Debt	641519	1424486	122%
Coupon	54765.06	140210.9	156%
Requared return on debt	8.54%	9.84%	15%
Yeild spreads	0.14%	1.44%	955%
Total Firm Value	2254521	2335223	4%
V-Bankruptcy	318293.3	814902.7	156%
V-Bancruptcy/ Total Firm Value	14%	35%	147%

Table 3.3.9 The optimal leverage rate for Rosneft 2010

As it can be seen from this table, the theoretical optimal leverage structure is the same as it is recommended to the Lukoil in 2010. This can be explained by the fact that the companies perform in a really similar way, in 2010. Nevertheless, the interesting difference here between Rosneft and Lukoil case in 2010 is that even increasing the leverage for more significant amount (114% for Rosneft versus 96% for Lukoil), the gains of this operations in terms of total firm value are less, then those for the Lukoil case (4% against 6%), so most likely, the reasons why the model here is over valuating debt are the same as for Lukoil. The model just does not take into account some risks, except currency risk and default risk. The similar situation holds only

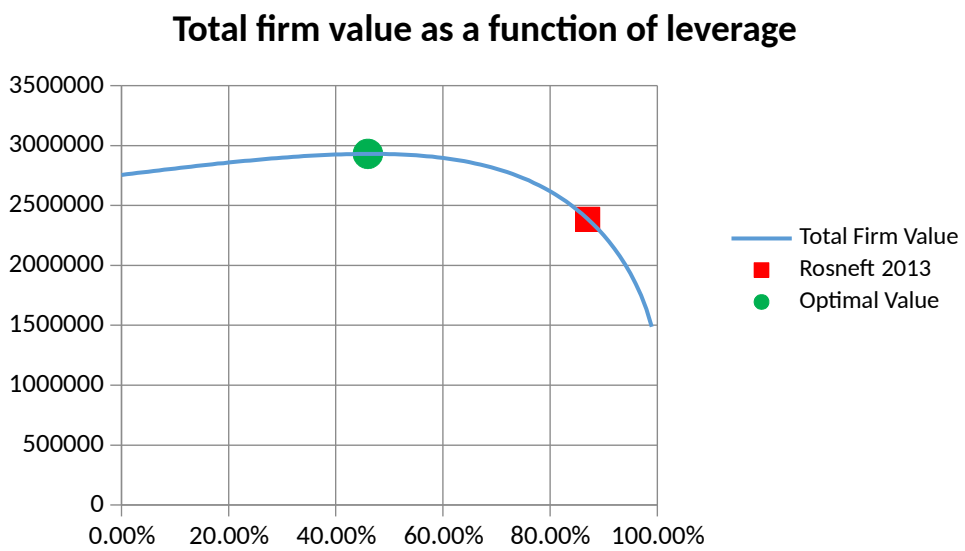
for the year 2010, while for other years it differs a lot in terms of model implementation in Lukoil and Rosneft case, so now the following years should be analyzed in order to gain better understanding of the model performance.

For the year 2013 the situation for Rosneft had changed significantly, unlike in Lukoil case. For example, the yield curve is moved to the following form:



Graph 3.3.11 Yield spreads on debt as a function of leverage

Here, the cost of debt curve is similar to those of the Lukoil for the same time period. However, it seems that the managers of the Rosneft decided to take significantly more debt, compared to the 2010 year. Basically, it is shown that in 2013, according to the model the company is over levered, and has more risk that it should have been taken In terms of possible total value of the firm, with the probability of the default the situation is following:



Graph 3.3.12 Total firm value as a function of leverage

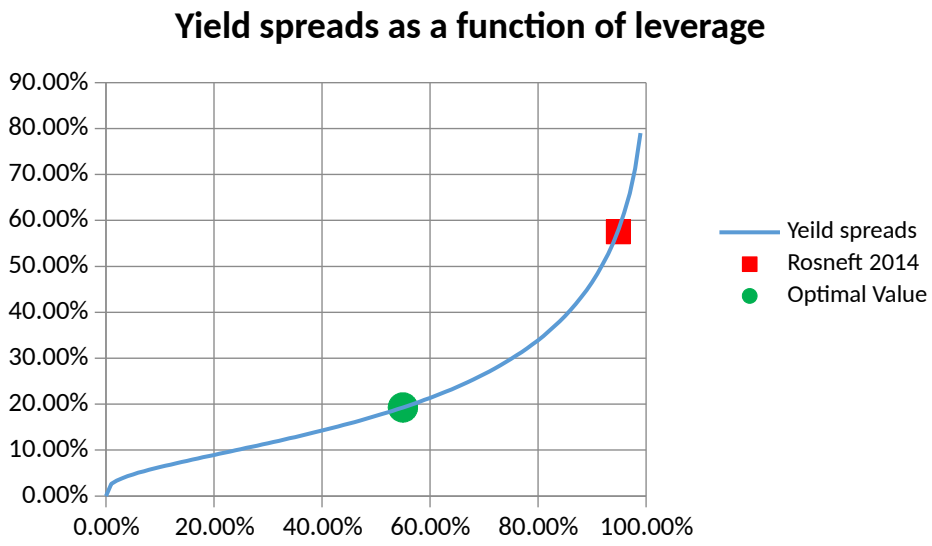
So, again, the theoretical leverage structure that should be in the Rosneft company, according to the model is nearly 50%, having the same amount as the Lukoil in the same time period. But, unlike the Lukoil, Rosneft seems to be over levered even in the pre-crisis years, when the Lukoil had too low leverage according to the model.

Values	*RosNeft 2013*	*Optimal Value*	*Change*
Leverage	87%	46%	-47%
Debt	2076000	1348282	-35%
Coupon	323249.4	120602.5	-63%
Requared return on debt	15.57%	8.94%	-43%
Yeild spreads	15.57%	1.51%	-90%
Total Firm Value	2384917	2931047	23%
V-Bankruptcy	1853560	691552.5	-63%
V-Bancruptcy/ Total Firm Value	78%	24%	-70%

Table 3.3.10. The optimal leverage rate for Rosneft 2013

Here, it can be seen, that according to the model, company has more risks, that it should take. The value of asset that triggers bankruptcy is almost 80% from the total value of the firm, which, with considered fact that the company has quite volatile business, seems to be to much risk to take. So here, the model suggest to reduce the value of leverage, and unlike all previous suggestions that will lead to slight increase in the firms value (around 5%), in this case the reduction of the risk taken by the company improves the value of the firm significantly for 23%. So, after the analisis it can be stated, that Rosneft case is not really corresponding to the models assumptions, since the predicted value of leverage is significantly less then observed one. From the other point of view, every management decision have some hidden partners, that cannot be seen from the external view of the company, and it might happened that Rosneft managers had some plan to use this high leverage in order to make more profits. However, the crisis appears in the next years, and it can be assumed, that high leverage policy, excepted by Rosneft was not the best path to take for the company development.

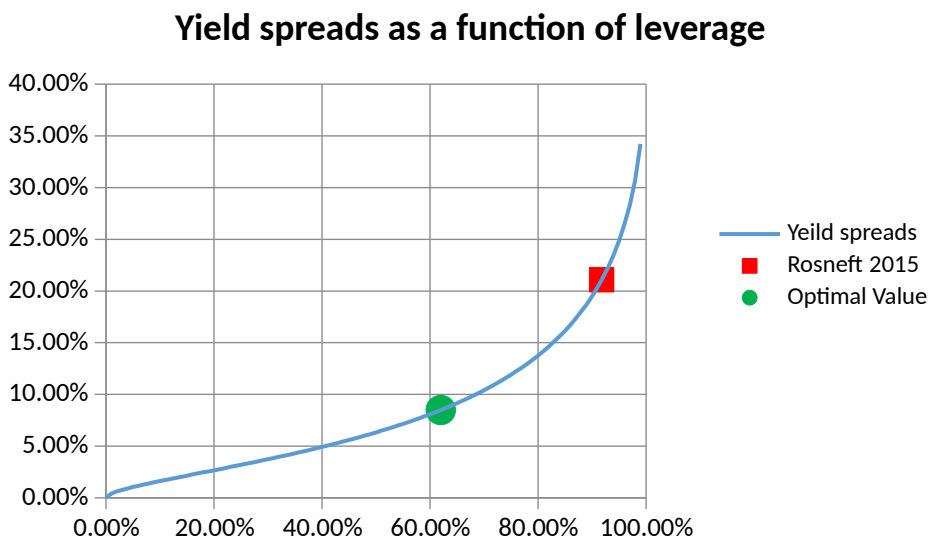
For the first crisis year, the situation for yield curve for the debt of the Rosneft company was following:



Graph 3.3.13 Yield spreads on debt as a function of leverage

In the first crisis year, the cost of debt for the Rosneft had been rose significantly. This is a result of a higher risks, associated with the company, due to high currency risks, and high risk of default. High leverage value in the pre-crisis years led to the debt reprising, and really high risks of the company, so the reared rate of return on debt become numerously high. This also can be explained with the fact that in 2014 the business value of the company fell, respectively to the pre-crisis years, and it was previously discussed, how dangerous this situation can be for the company. In order to make some conclusions it seems reasonable to compare it to the values in 2015.

The yield spreads for Rosneft in 2015 are following:

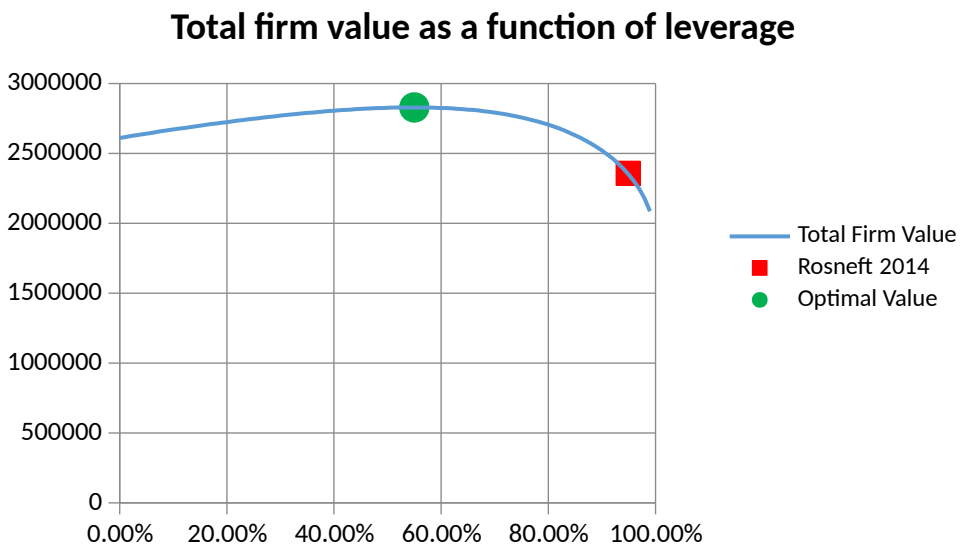


Graph 3.3.14 Yield spreads on debt as a function of leverage

The main difference here is that the debt become slightly “cheaper” for Rosneft in 2015. This can be explained by the facts, that firstly, the currency risk is less valuable in this year.

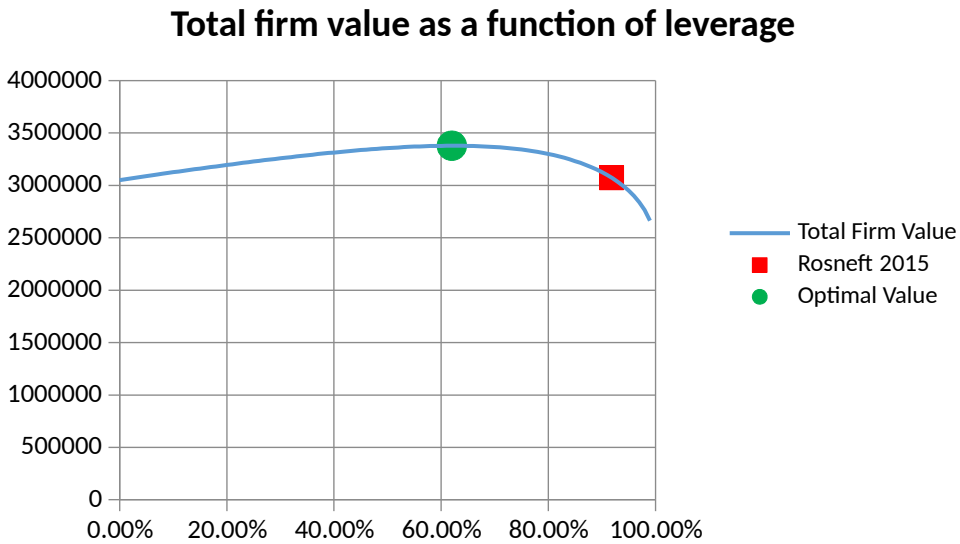
Secondly, as it was described previously, the firm had increased its value of business, compared not only to previous year, but even for the pre-crisis years. However, the firm is still quite over levered, from the point of view of proposed model.

The differences for the crisis years in terms of value of the company are following:



Graph 3.3.15 Total firm value as a function of leverage

And for the year 2015 there is a slightly different situation:



Graph 3.3.16 Total firm value as a function of leverage

From the analysis of this graphs, it can be seen, that for 2015 the situation in terms of risks is slightly better, then in 2014, the reasons for that are the same as the reasons for difference in the yield curves. The numerical differences are following:

Values	*RosNeft 2014*	*Optimal Value*	*Change*
Leverage	95%	55%	-42%
Debt	2240000	1556203	-31%
Coupon	1512577	486520.5	-68%
Requared return on debt	67.53%	31.26%	-54%
Yeild spreads	55.55%	19.28%	-65%
Total Firm Value	2359937	2829460	20%
V-Bankruptcy	1867223	600592.4	-68%
V-Bancruptcy/ Total Firm Value	79%	21%	-73%

Table 3.3.11 The optimal leverage rate for Rosneft 2014

And for the 2015:

Values	*RosNeft 2013*	*Optimal Value*	*Change*
Leverage	87%	46%	-47%
Debt	2076000	1348282	-35%
Coupon	323249.4	120602.5	-63%
Requared return on debt	15.57%	8.94%	-43%
Yeild spreads	15.57%	1.51%	-90%
Total Firm Value	2384917	2931047	23%
V-Bankruptcy	1853560	691552.5	-63%
V-Bancruptcy/ Total Firm Value	78%	24%	-70%

Table 3.3.12 The optimal leverage rate for Rosneft 2015

In both cases, the Rosneft considered to be over levered and bearing too many risks. The model predicts, that lowering the leverage value to the fraction of approximately 50% will allow to increase the company's overall firms value for nearly 20%. It is necessary to state, that optimal leverage value for Rosneft in the crisis years is quite similar to the optimal leverage value of Lukoil at the same dates. To sum up, the overall model predictions are very similar fro the Lukoil and Rosneft for the same time periods. However, unlike Lukoil Company, Rosneft didn't reach the optimal capital structure predicted by the model, and consider to be over levered, especially in the crisis years. The other interesting outcome is that, while the under – levered firms can gain only small amounts of additional firm value by taking more debt, however, the companies with over-levered structure, gaining significantly more value, by reducing the risks.

Finally, it can be stated, that the assumptions of time-independence of most of the model parameters holds, if the situation on market does not change significantly.

The final part of the model testing, is to test on t6he company, that is from the one point of view, similar to the Lukoil and Rosneft business, but at the same time will be able to keep the same amount of leverage for pre-crisis and crisis years. This approach is necessary in order to understand, how the model performs for the companies, that are staying in the relatively same situation even in the crisis, that effects the other analyzed companies, since previously it was found out that the model offer the similar solutions for the alike market situations. So, by testing the model on such a company, the following questions can be answered: if the assumption of time-independence of the model parameters holds in certain circumstances, and if the model over or under valuating the level of leverage in the companies. To fulfill this goal the company Novatek was chosen.

3.3.3 Case of Novatek company

The Novatek case in this work is used as a litmus test, in order to understand the performance of the model from the point of view of over or under valuating the leverage fraction, under the same circumstances. So, unlike the previous companies, the Novatek case will be analyzed as a whole, without single year situation description. So, as well as for the previous cases, the first step of model implementation algorithm should be used and the proxies for the Nivatek Company should be described. As well as in the previous cases, the data was gained from the reports of the company and open sources, like bank of Russia, and market information. The proxies for the Novatek are:

Novatek Company	Year			
Proxies	*2010*	*2013*	*2014*	*2015*
Value of Debt (mln rubles)	72199	165621	245679	350645
Risk-free rate	8.40%	7.43%	11.98%	11.35%
Value of equity (mln rubles)	440936.71	916397.01	930434.1	1422223.7
Year Volatility of the company	30.99%	24.17%	38.91%	54.10%

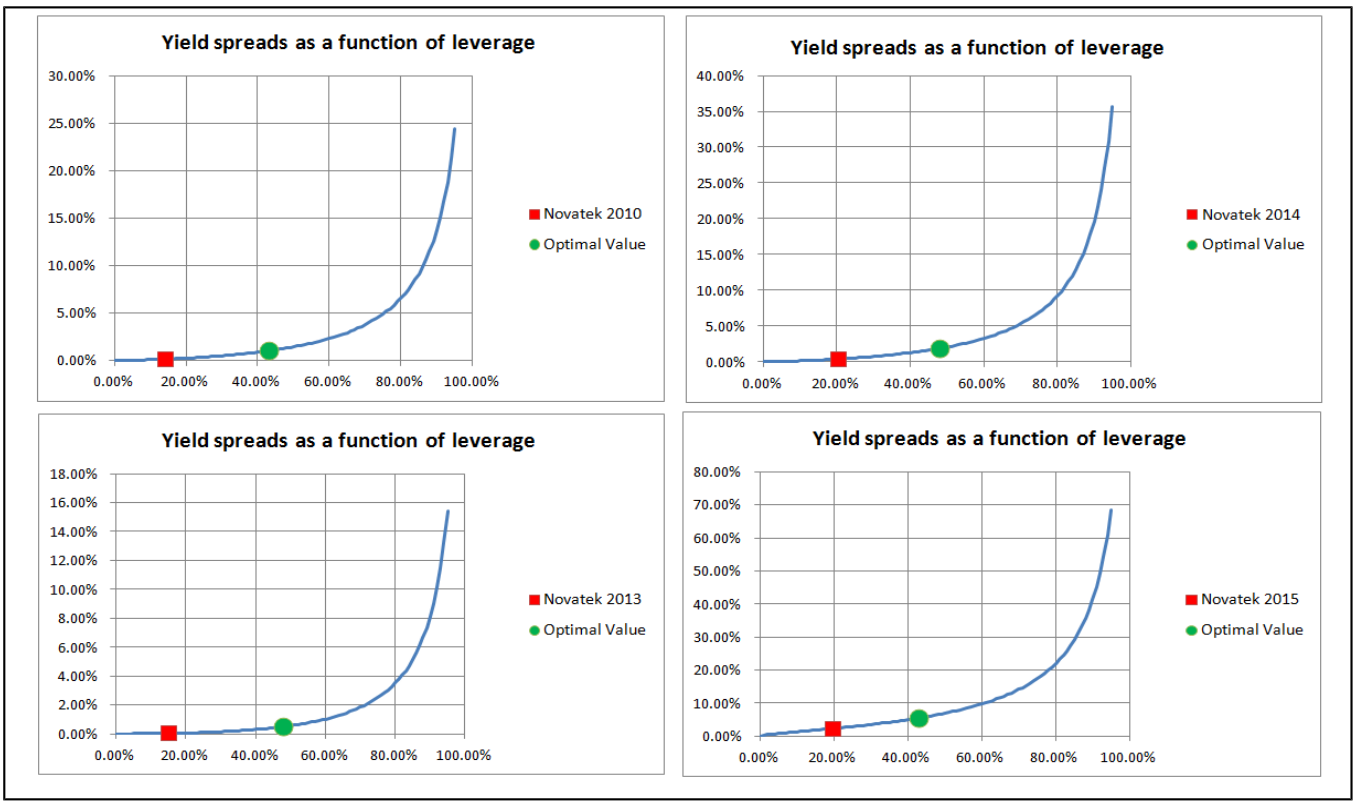
Table 3.3.13. The proxy values for Novatek company

It can be already seen, that there is no huge “jump” in the values, like it were in the previous cases in the first crisis year. The growth of the values seems to be less sharply, and the assumption can be made, that all the values have similar fractions to each other in all years, and it can be illustrated better, after implementation the following steps of model’s algorithm, and defining the estimated parameters:

Novatek Company	Year			
Estimated Parameters	*2010*	*2013*	*2014*	*2015*
Value of business	502349.41	1053474	1130465.713	1713342
Volatility of business	32.58%	25.39%	41.03%	57.57%
Rate of recovery	87.50%	87.78%	87.00%	87.90%
Effective Tax Rate	18.14%	19.82%	27.00%	24.51%
Risk free rate	8.40%	7.43%	11.98%	11.35%
Coupon	6343.0314	12718.4	35199.90441	48766.71
Requared return on debt	8.79%	7.68%	14.33%	13.91%

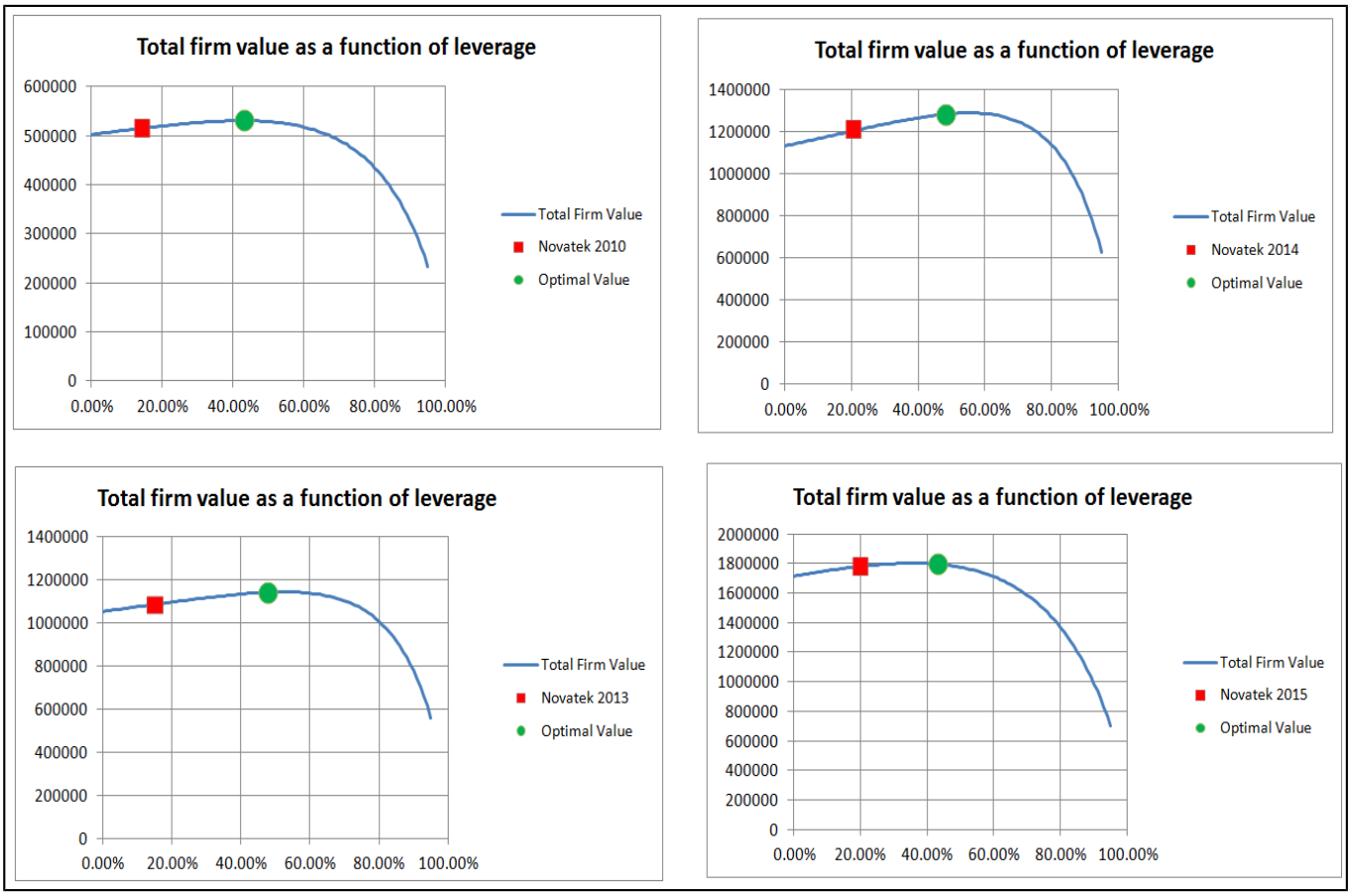
Table 3.3.14. Estimated parameters for the Novatek company

So, as it can be seen, the values are not growing as rapidly as in the previous cases, and, the most importantly, the "jump" in volatility for the 2014 year is not so huge as it were for the Lukoil and Rosneft cases. Also, the value of business of the company is constantly growing, demonstrating the "normal" case of the company performance. Required return on debt demonstrates the same trends as were found in the cases of the Lukoil and Rosneft, with increasing in the 2013, and after this slightly falling for 2015, however, the difference between this values is not as significant as it were for the previous companies. So, all in all it should be mentioned that the Novatek company is similar to the other estimated cases in terms of parameters changes and overall patterns. Now, the performance of the company thought the years should be analyzed, to show difference in model approach. So, the yields for the different years are presented in the following graph.



Graph 3.3.17. Yield spreads on debt as a function of leverage for 2010-2015

In this graph can be seen that through the years the cost of debt changes, however the leverage structure in observed in the company, as well as the optimal capital structure predicted by the model stays nearly the same. This situation is caused by the fact, which basically, the Novatek is not suffering from the crisis as strong as the other analyzed companies. The shape of the curve also changes trough the years, since more costly – or risky in other words - the debt becomes, faster the yield spreads grow with the value of leverage. However, it does not affect the company's optimal and observed optimal capital structure, since the internal risk of the firm is following the market trend perfectly. This means, that Novatek, unlike Lukoil or Rosneft is not creating additional risk by itself, with the over levered capital structure, like for example Rosneft. The performance of the model in terms in overall company value as a function of leverage is presented on the following graph.



Graph 3.3.18. Total firm value as a function of leverage for 2010 - 2015

The graph represent the situation of model implementation, where the crisis influence both calculated and observed capital structure of the firm in quite a minor way. The approximate optimal leverage value is nearly 40- 45 %, and the observed one is around 20% for all the years. However, the overall increase in riskiness can seen on this graphs. Through the years, with the appearance of the crisis, the "sharper" fall of the company's value appears at the huge leverage

values. This fact is understandable, since, in more risky environment the company operates, the more risky it is to have huge leverage values, as it was shown on the Rosneft example. Moreover, it can be said that most of the time-independence assumptions cal be called realistic, since it was shown, that if the company doesn't change a lot with the market changes, the calculated values of optimal capital structure are quite the same for all the years, regardless of crisis, and it is true also for the observed values. Finally, it can be concluded, that in all the years the model over valuates the real value of the capital structure, which can be explained by the model limitations in the risk valuation.

3.4 Analysis of the results and the managerial implications

Application of the proposed model showed some difference in the calculated level of leverage and observed one. For the most of cases, the calculated level of leverage exceeds the observed level of leverage. This debt value overvaluation was already mentioned in this type of models by the previous researches [Taxeira, 2007]. This can be explained by the fact, that the proposed model, mostly concentrates on the value of default risk. However, this is not only type of risk that company suffers from. In the model modification, proposed in this thesis, the currency risk of the debt was also taken into account. Nevertheless, the value of risks is still under valuated by the model, since firstly, the proposed method of the valuing currency risk, assumes that the currency returns volatility will remain relatively constant through time, but as it was shown by the implementing of the model, this situation is not really happening, especially, when the crisis appears in the country, where the company operates. So, the first recommendation of the further model development will be, the modification of the proposed model due to more reliable analysis of the debt reprising due to the currency risk change. The other reason for debt value overvaluation is a really specific debt approach in the theoretical model. The assumption here is that the debt of the company can be presented with the perpetual coupons debt, so the bankruptcy will be associated with the inability of the firm to pay required coupons. However, this assumption might not be hold in some practical situation, so the second theoretical recommendation is to develop a model in a way that will be similar to the Leland and Toft model (1996), since as it was mentioned before, due to limited recourses of the Master thesis recourses, I have made a decision to base the work on the model that was more suitable for the practical implementation. Nevertheless, the algorithm developed in the current work and modifications that were made for the theoretical model, can be called universal for the implementation of the models based on the "option like" approach to the company's equity, so the further research, based on this thesis might be – the described algorithm implementation to the other theoretical models.

From the other point of view, the model implementation was quite successful for some cases – like Lukoil Company in 2014- 2105 years, since the calculated capital structure corresponded perfectly to the observed results. Moreover, the analysis shows, that in the stable environment most of the model assumptions holds, and the parameters that were assumed to be time-independent are time independent under certain circumstances. In addition to that, the model testing on the real data showed that most of the gained results are the same as were expected, and model does not provide any "unordinary" or non logical solutions. So, it seems that after several modifications the model might become quite powerful tool in identifying optimal leverage structures for the different companies.

The theoretical contribution of this thesis can be described as, firstly this thesis one of the rare examples of the implementation of the models based on the "option-like" nature of equity to the real companies situation. Even thought, certain limitation of the model was found, it could be stated that the implementation of the model showed quite reasonable results, and the model itself seems to be quite applicable to the real business situations. Another important theoretical contribution of the work is the identification of the narrow places of the implementation of this type of models to the real cases. Main recommendations connected to this narrow places are- to include more risk estimators in the model, and add some type of time-dependence and design the model that consider the changes of the variables trough time, and some trends, that can affect the leverage structure of the company. Also, it would be valuable to add some parameters, based on the companies specific strategies, because the business model of the company and managerial decisions connected to it seem to have an effect to the leverage analysis. So, the further development possibilities of the proposed model were defined, and well as the possibilities of its' further modifications. The algorithm developed in this paper can be used as a basis for the future research related to the implementation of the similar models since, the designed algorithm can be called universal for this type of models.

Managerial implications of the model seem to be very broad. Firstly, this model will be interesting for the company's management, in order to optimize its leverage structure. Moreover, the company's managers can analyze the situation and gain the understanding which level of assets of the firm is required, if they chose the type of debt, described in the model. Since, the company's management have some internal information, they can modify a model in a way, that it's performance will be much more accurate for the specific firm. Also, with this model the tax benefits of the company, with the respect of the default risk, can be defined, so the company's management will have a clear understanding of the possible debt benefits. With the help of the model, the value of unlevered firm can be found as well as the volatility of it, so the analysis of the company performance in terms of value creation can be understood by the strategic

department of the company managers. In addition to that, by using the model, the managers of the company can identify what theoretical value of assets would trigger bankruptcy, and comparing it to the value of the firm the measurement of riskiness of the position that the company has can be understood, so some steps in order to reduce risks might be taken. For example, if the managers of the Rosneft Company in 2013 would use the model to analyze the company, they would found out that the firm is over levered, and most probably will reduce the value of leverage, so in 2014 the crisis will not affect the Rosneft in a way it did. Finally, the managers of the company can gain better understanding of the required return on debt for the certain company in different market situations, so their decisions about the company's leverage will be more accurate, and with the changing the capital structure of the company, they will bring additional value to the shareholders , since the market value of the company will rise – due to higher tax benefits, like in Lukoil and Novatek cases, or due to risk of default will be reduced, like in the Rosneft case.

On the other hand, the model can be used by debt holders and the equity holders of the company. From the point of view of both groups, it would be quite important to understand the level of the default risk of the company. For the debt holders, it would be interesting to identify the recovery rate for the certain company, with the proposed model, since it is the amount that they will gain in case of the default. Moreover, with the help of the model debt holders can understand, what amount of debt returns should the company pay, with the respect of its' default risk. Also, the volatility of unlevered firm can be found with this model, which is important measurement for both, equity and debt holders, since it identifies the future possible change of the value of firms business. The debt holders also can use the results of the model implementation, as a clue of the existence of the agency problem. If the calculated results differs a lot from the expected ones, while most of the assumptions of the model seems to be holding, it can be a signal of the appearance of the "asset substitution" problem, so the debt holders should be more concerned about their relations with the company. For the equity holders the model can be valuable it terms of identifying the unlevered firm value, and what is more important it changes through the time. Since, the firms value, and equity value depends heavily on the business value of the firm, owners of the company have to pay a lot of attention to its changes. If the value of the unlevered firm is falling, it is a clear evidence that the management of the firm is not efficient enough, and the owners of the company might conceder the possibility of changing firms management, or even to sell their part of the company. Also, since the equity holders gain value from share prices rising, it would be interesting for them to see, how the market capitalization of the company can be increased with the changing the capital structure. So, if it was found out, that the assumptions of the model holds, but the total firms value can be increased

with the change of leverage, than, from the point of view of the owners of the company, management of the company is not using an opportunity of increasing the value for the shareholders.

All in all, the model developed in this master thesis, from a theoretical point of view uses more accurate analysis of the company capital structure, then commonly used models – like modifications of Hamada's equation. From the theoretical point of view, this master thesis has some value, due to implementation of the model based on the "option-like" approach to equity with defining the "narrow places" of this approach, and designing the algorithm of implementing similar type of models to the real cases, which opens huge possibilities for further studies on the same topic. From the practical point of view, as it was described previously, since the capital structure is such an important side of the firms performance, the model that describes the main patterns of the firms leverage and recommends the optimal structure of it, has a huge variety of managerial implications and is interesting for the different stake holders of the company.

CONCLUSION

In order to understand the nature of the problem of identifying the optimal capital structure, it is important to state that corporate capital structure and value of debt are interlinked variables. The debt value, because it depends on yield spreads, also depend of level of leverage, because higher leverage level, means more risk in the company, and higher risk means higher yields. So, the identification of the optimal capital structure is quite a tricky problem to solve.

The models of identifying the optimal capital structure can be divided to the two big groups. The first one, are the models based on the Modigliani-Miller's theory, and they can be called classical models. However, despite the fact that classical optimal structure models are widely used and discussed in academic papers, they often require unrealistic assumptions that make them not practically valuable. That is why this paper is focused on the other type of optimal capital structure models defining equity as an option. This type of models, first, seems to be more realistic that enables to make practical conclusions.

As a basis model, were chosen the model developed by Hayne Leland. However, in order to apply this model to the task of identifying the optimal capital structure for the companies some modifications were made. As it was already stated, the model invented by Hayne Leland was chosen, on the one hand due to its' positive sides, and close to reality approach, and in the other hand, because of its simplicity.

The research goal can was stated as developing the optimal debt structure identification model. This goal was achieved by achieving all the thesis objectives. After the creation of the model the algorithm of its' implementation to the real companies were designed. Finally, the model was tested on the real companies under different market conditions. And practical recommendations to those companies were given.

Application of the proposed model showed some difference in the calculated level of leverage and observed one. For the most of cases, the calculated level of leverage exceeds the observed level of leverage. This debt value overvaluation was already mentioned in this type of models by the previous researches. However, the recommendations to solve this problem were given in the research.

All in all, the model developed in this master thesis, from a theoretical point of view uses more accurate analysis of the company capital structure, then commonly used models – like modifications of Hamada's equation. Also, this master thesis has contributes the scientific society, due to implementation of the model based on the "option-like" approach to equity with defining the "narrow places" of this approach, and designing the algorithm of implementing similar type of models to the real cases, which opens huge possibilities for further studies on the

same topic. From the practical point of view, as it was described previously, since the capital structure is such an important side of the firms performance, the model that describes the main patterns of the firms leverage and recommends the optimal structure of it, has a huge variety of managerial implications and is interesting for the different stake holders of the company. However, it can be said that the model needs further development in a way that it was recommended, in order to gain more accurate results.

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